

WYKŁAD 4

Wstęp do optyki i fizyki materii skondensowanej

Optyka

Mariusz Semczuk

msemczuk@fuw.edu.pl

Fizyka materii skondensowanej:

Johannes Binder

Oddziaływanie atomu z kwantowym polem E-M

Stany ubrane

Stany własne *atom + pole* bez oddziaływania $|i\rangle_A \otimes |\bar{n}\rangle_F \equiv |i, n\rangle$

$$\hat{H}_{JC} = \hbar\omega_c \hat{a}^\dagger \hat{a} + \frac{1}{2} \hbar\omega_a \hat{\sigma}_z + \hbar\Omega(\hat{\sigma}_+ \hat{a} + \hat{\sigma}_- \hat{a}^\dagger)$$

Dla atomu dwupoziomowego bez oddziaływania rozważmy stany $|2\rangle|n-1\rangle$ i $|1\rangle|n\rangle$

$$\left(\hbar\omega \hat{a}^\dagger \hat{a} + \frac{\hbar\omega_{21}}{2} \hat{\sigma}_z \right) |2, n-1\rangle = \left((n-1)\hbar\omega + \frac{1}{2}\hbar\omega_{21} \right) |2, n-1\rangle$$

$$\left(\hbar\omega \hat{a}^\dagger \hat{a} + \frac{\hbar\omega_{21}}{2} \hat{\sigma}_z \right) |1, n\rangle = \left(n\hbar\omega - \frac{1}{2}\hbar\omega_{21} \right) |1, n\rangle.$$

$$\hat{\mathcal{H}}_{JC} = \hbar \begin{pmatrix} (n-1)\omega + \frac{1}{2}\omega_{21} & -ig\sqrt{n} \\ ig\sqrt{n} & n\omega - \frac{1}{2}\omega_{21} \end{pmatrix}$$

$$g = 2\Omega$$
$$\Omega = -\frac{\mu}{\hbar} \sqrt{\frac{\hbar\omega_c}{\epsilon_0 V}} \sin(kz)$$

Oddziaływanie atomu z kwantowym polem E-M

Stany ubrane

Dla atomu dwupoziomowego bez oddziaływania rozważmy stany $|2\rangle|n-1\rangle$ i $|1\rangle|n\rangle$

$$\hat{\mathcal{H}}_{JC} - \left(n - \frac{1}{2}\right) \hbar\omega \hat{\mathbb{I}} = \frac{\hbar}{2} \begin{pmatrix} \Delta & -i2g\sqrt{n} \\ i2g\sqrt{n} & -\Delta \end{pmatrix} \quad \Delta = \omega_{21} - \omega$$

Wartości własne macierzy: $\lambda_{\pm} = \pm \frac{\hbar}{2} \sqrt{\Delta^2 + 4g^2 n}$ $\Omega_n = \sqrt{\Delta^2 + 4g^2 n}$

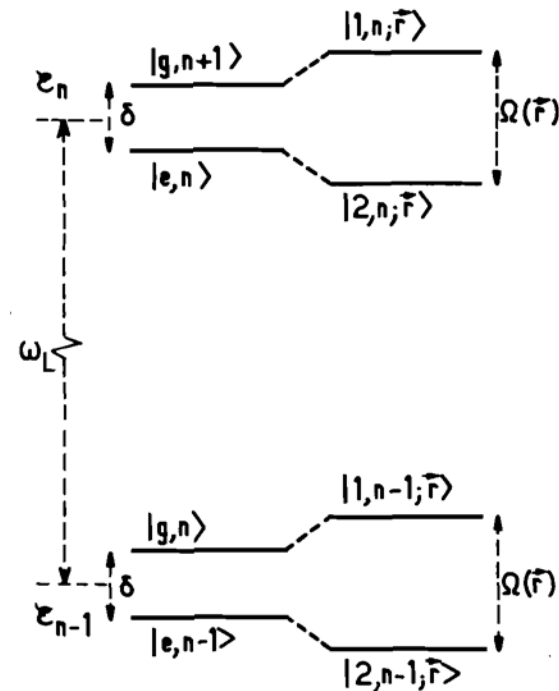
Energie własne całego układu: $W_{n,\pm} = \left(n - \frac{1}{2}\right) \hbar\omega \pm \frac{\hbar}{2} \Omega_n$

Stany własne (stany ubrane, ang. dressed states)

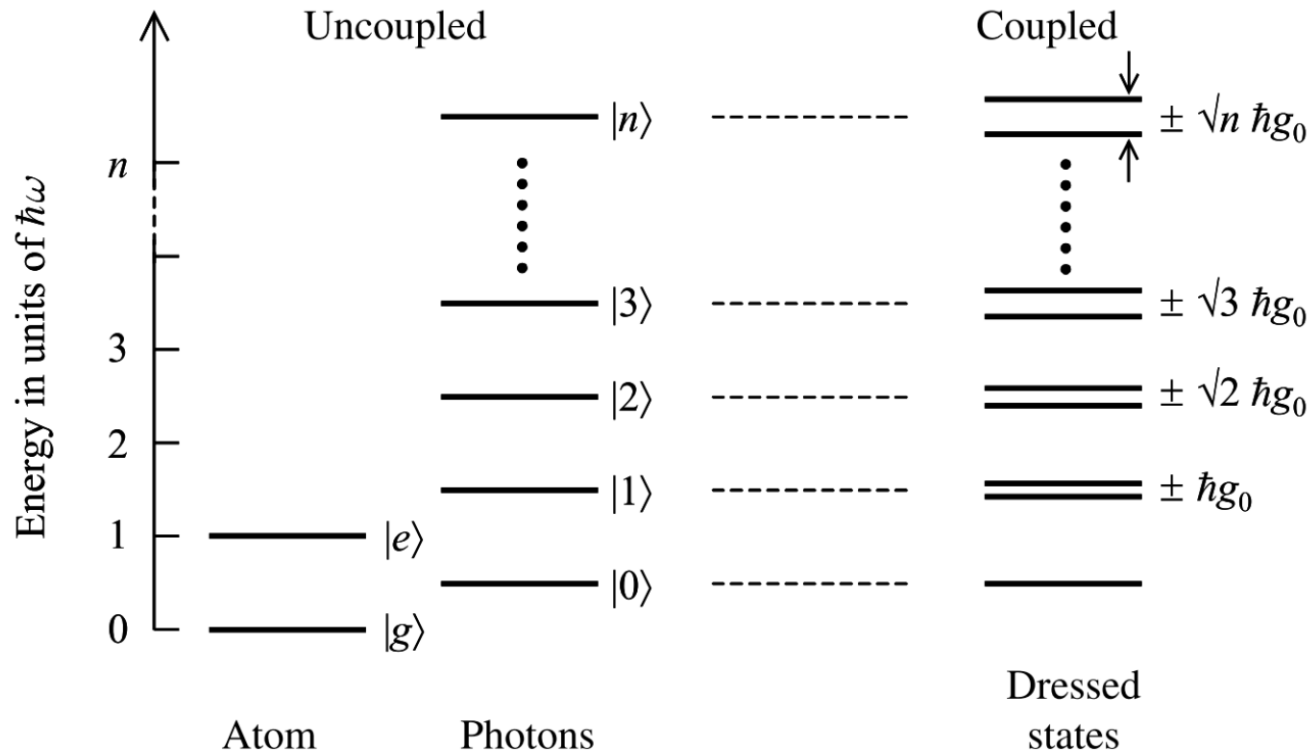
$$\begin{aligned} |n, +\rangle &= \cos \phi_n |2, n-1\rangle + \sin \phi_n |1, n\rangle \\ |n, -\rangle &= -\sin \phi_n |2, n-1\rangle + \cos \phi_n |1, n\rangle \end{aligned}$$

$$\cos \phi_n = \sqrt{\frac{\Omega_n + \Delta}{2\Omega_n}}, \quad \sin \phi_n = \sqrt{\frac{\Omega_n - \Delta}{2\Omega_n}}$$

$$\operatorname{tg} 2\phi_n = \frac{2|g|\sqrt{n}}{|\Delta|}$$



Stany ubrane w modelu Jaynesa - Cummingsa



Użyto bazy $\{|g, n\rangle, |e, n - 1\rangle\}$ aby $\Omega_n = \sqrt{n} \hbar g_0$.

Oddziaływanie atomu z kwantowym polem E-M

Słabe oddziaływanie ($2|g|\sqrt{n} \ll |\Delta|$), kwadratowy efekt Starka

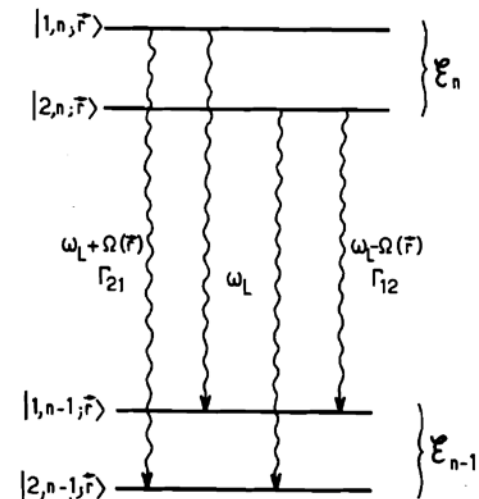
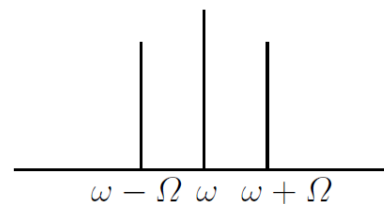
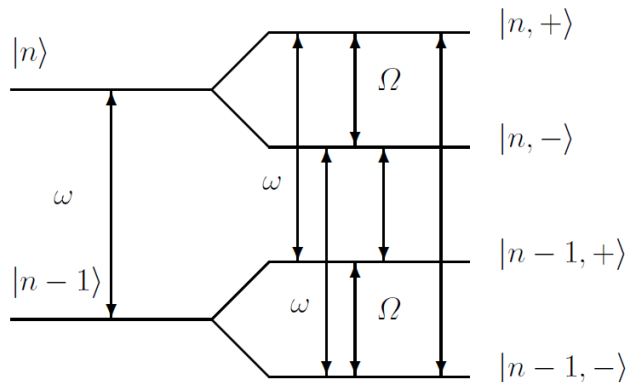
$$\Omega_n \approx |\Delta| \left(1 + \frac{1}{2} \frac{4g^2 n}{\Delta^2} \right)$$

$$W_{n,+} = \left(n - \frac{1}{2} \right) \hbar\omega + \frac{\hbar}{2} |\Delta| \left(1 + \frac{1}{2} \frac{4g^2 n}{\Delta^2} \right) = (n - 1) \hbar\omega + \frac{\hbar}{2} \omega_{21} + \hbar \frac{g^2 n}{|\Delta|}$$

$$W_{n,-} = \left(n - \frac{1}{2} \right) \hbar\omega - \frac{\hbar}{2} |\Delta| \left(1 + \frac{1}{2} \frac{4g^2 n}{\Delta^2} \right) = n \hbar\omega - \frac{\hbar}{2} \omega_{21} + \hbar \frac{g^2 n}{|\Delta|}.$$

Silne oddziaływanie ($2|g|\sqrt{n} \gg |\Delta|$), liniowy efekt Starka

$$W(n, \pm) = \left(n - \frac{1}{2} \right) \hbar\omega \pm \hbar|g|\sqrt{n}$$



Struktura subtelna

Energia atomu w modelu Bohra

$$E_n = - \frac{\mu e^4}{2\hbar^2} \frac{1}{(4\pi\epsilon_0)^2} \frac{Z^2}{n^2} = \frac{\mu c}{2\hbar} \frac{(Z\alpha)^2}{n^2}$$

Struktura subtelna: spin elektronu + poprawki relatywistyczne+człon Darwina

$$H = H_0 + H_{\text{kinetic}} + H_{S-O} + H_{\text{Darwinian}}$$

$$H_{\text{kinetic}} = -\frac{p^4}{8m^3c^2} \rightarrow \boxed{E_{\text{kinetic}} = -\frac{E_n^2}{2mc^2} \left(\frac{4n}{l + \frac{1}{2}} - 3 \right)}, \quad E_n = \frac{e^2}{8\pi\epsilon_0 a_0 n^2}$$

$$H_{\text{Darwinian}} = -\frac{\hbar^2}{8m_e^2 c^2} 4\pi \left(\frac{Ze^2}{4\pi\epsilon_0} \right) \delta^3(\vec{r}) \rightarrow \langle H_{\text{Darwinian}} \rangle = -\frac{\hbar^2}{8m_e^2 c^2} 4\pi \left(\frac{Ze^2}{4\pi\epsilon_0} \right) |\psi(0)|^2$$

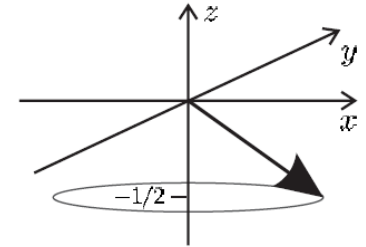
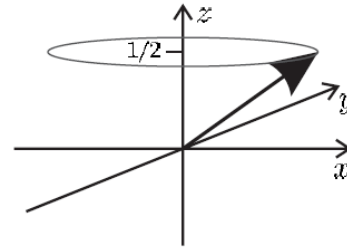
$$\boxed{E_{\text{Darwinian}} = \frac{2n}{m_e c^2} E_n^2 \text{ dla } l = 0}$$

Struktura subtelna

Funkcja falowa ze spinem $\Psi = R_{n,l}(r) Y_{l,m}(\theta, \phi) |s m_s\rangle = R_{n,l}(r) |l m_l s m_s\rangle$

Spin góra/dół ($|s| = \sqrt{s(s+1)}\hbar$)

$$|s = \frac{1}{2}, m_s = \frac{1}{2}\rangle \equiv |\uparrow\rangle$$



Odziaływanie spin - orbita

Elektron poruszający się w polu elektrycznym jądra $\mathbf{B} = -\frac{1}{c^2} \mathbf{v} \times \mathbf{E}$

$$\mathbf{E} = \frac{1}{e} \frac{\partial V}{\partial r} \frac{\mathbf{r}}{r} \quad \mathbf{B} = \frac{1}{m_e c^2} \left(\frac{1}{e r} \frac{\partial V}{\partial r} \right) \mathbf{r} \times m_e \mathbf{v} = \frac{\hbar}{m_e c^2} \left(\frac{1}{e r} \frac{\partial V}{\partial r} \right) \mathbf{l} \quad \hbar \mathbf{l} = \mathbf{r} \times m_e \mathbf{v}$$

$$H = -\boldsymbol{\mu} \cdot \mathbf{B} = g_s \mu_B \mathbf{S} \cdot \frac{\hbar}{m_e c^2} \left(\frac{1}{e r} \frac{\partial V}{\partial r} \right) \mathbf{l} \quad \boldsymbol{\mu} = -g_s \mu_B \mathbf{S}$$

$$g_s \simeq 2 \quad \mu_B = e\hbar/2m_e$$

Po uwzględnieniu precesji Thomasa – efektu relatywistycznego związanego z obrotem układu odniesienia:

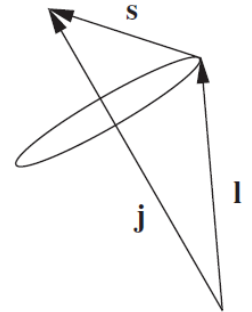
$$H_{s-o} = (g_s - 1) \frac{\hbar^2}{2m_e^2 c^2} \left(\frac{1}{r} \frac{\partial V}{\partial r} \right) \mathbf{s} \cdot \mathbf{l}$$

Struktura subtelna

Odziaływanie spin - orbita

Dla wodoru: $\frac{1}{r} \frac{\partial V}{\partial r} = \frac{e^2/4\pi\epsilon_0}{r^3}$ $E_{s-o} = \frac{\hbar^2}{2m_e^2 c^2} \frac{e^2}{4\pi\epsilon_0} \left\langle \frac{1}{r^3} \right\rangle \langle \mathbf{s} \cdot \mathbf{l} \rangle$

$$\left\langle \frac{1}{r^3} \right\rangle = \int_0^\infty \frac{1}{r^3} R_{n,l}^2(r) r^2 dr = \frac{1}{l(l + \frac{1}{2})(l + 1)} \left(\frac{Z}{na_0} \right)^3$$



Całkowity moment pędu $\mathbf{J} = \mathbf{l} + \mathbf{s}$ jest zachowany

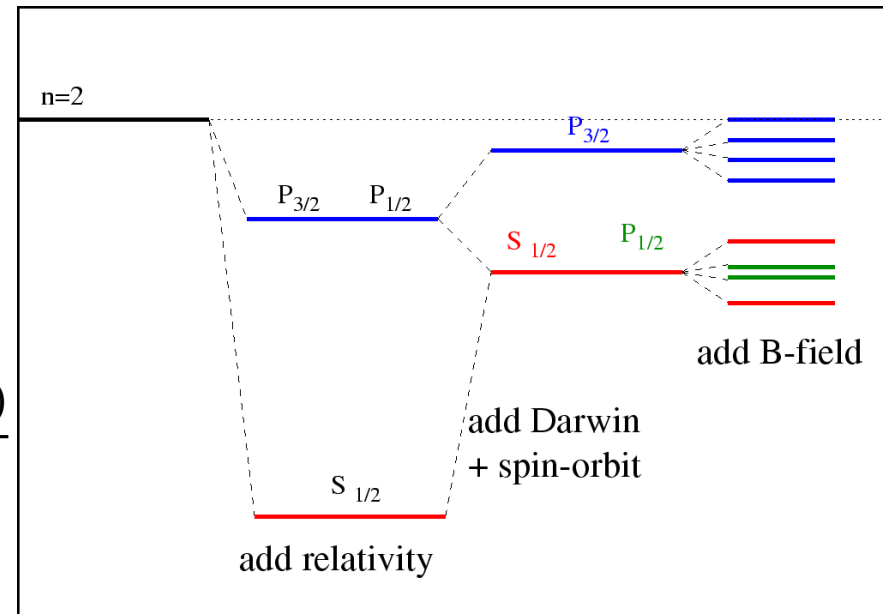
$$\langle \mathbf{s} \cdot \mathbf{l} \rangle = \frac{1}{2} \{j(j + 1) - l(l + 1) - s(s + 1)\}$$

$$E_{s-o} = \frac{\beta}{2} \{j(j + 1) - l(l + 1) - s(s + 1)\}$$

$$\beta = \frac{\hbar^2}{2m_e^2 c^2} \frac{e^2}{4\pi\epsilon_0} \frac{1}{(na_0)^3 l(l + \frac{1}{2})(l + 1)}$$

$$E_{s-o} = \frac{E_n^2}{m_e c^2} n \frac{j(j + 1) - l(l + 1) - s(s + 1)}{l(l + \frac{1}{2})(l + 1)}$$

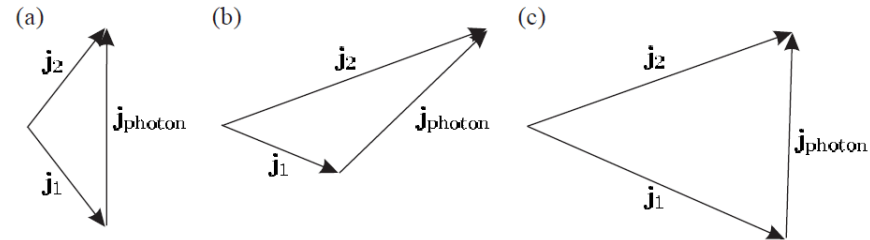
$$E_{s-o} \sim Z^4$$



Struktura subtelna

Przejścia między poziomami subtelnymi

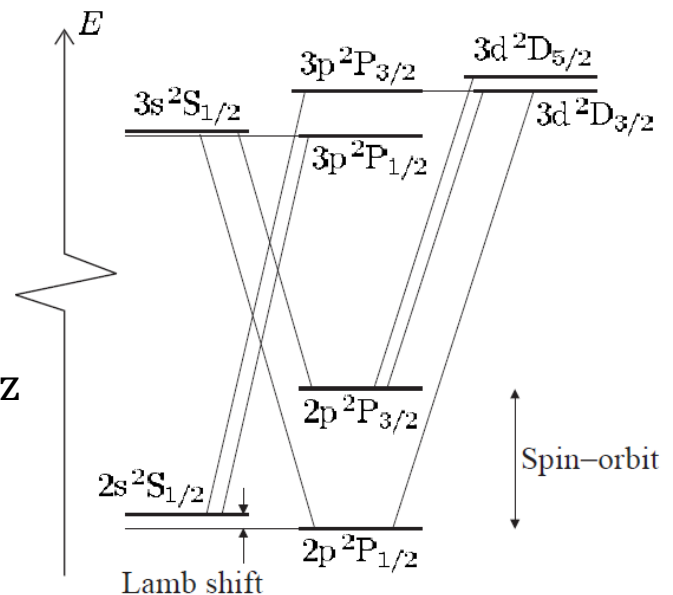
Electric dipole transitions	Magnetic dipole transitions
$\Delta J = 0, \pm 1$ (but not $J = 0$ to $J' = 0$)	$\Delta J = 0, \pm 1$ (but not $J = 0$ to $J' = 0$)
$\Delta M_J = 0, \pm 1$	$\Delta M_J = 0, \pm 1$
Parity change	No parity change
$\Delta l = \pm 1$	$\Delta l = 0$ } No change of
Any Δn	$\Delta n = 0$ } configuration
$\Delta L = 0, \pm 1$	$\Delta L = 0$
$\Delta S = 0$	$\Delta S = 0$



Przesunięcie Lamba (dla wodoru) - elektrodynamika kwantowa

$$\Delta E_{Lamb} = \alpha^3 \frac{1}{6\pi} \ln \frac{8}{\alpha^2} \approx 1.6 \text{ GHz}$$

Aktualnie zmierzona wartość: 1.058 GHz



Struktura subtelna

Sprężenie LS (Russella-Saundersa)

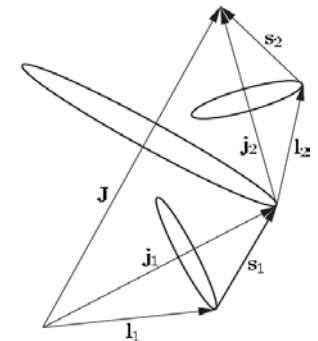
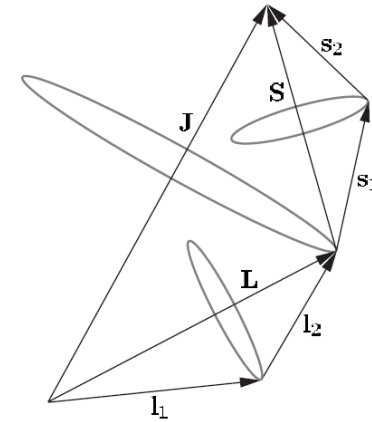
$$E_{s-o} = \beta_{LS} \langle \mathbf{S} \cdot \mathbf{L} \rangle \quad |LSJM_J\rangle$$

$$|lsjm_j\rangle = \sum_{m_l, m_s} C(lsjm_j; m_l, m_s) |l m_l s m_s\rangle$$

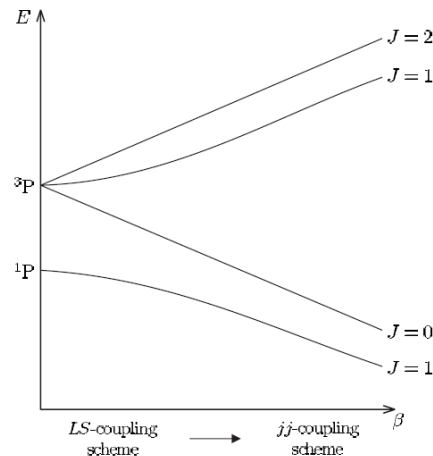
$$\mathbf{L} \cdot \mathbf{S} = (\mathbf{J} \cdot \mathbf{J} - \mathbf{L} \cdot \mathbf{L} - \mathbf{S} \cdot \mathbf{S}) / 2$$

$$E_{s-o} = \frac{\beta_{LS}}{2} \{ J(J+1) - L(L+1) - S(S+1) \}$$

$$\Delta E_{FS} = E_J - E_{J-1} = \beta_{LS} J$$



Sprężenie jj – słabe oddziaływanie pomiędzy elektronami



Struktura nadsubtelna

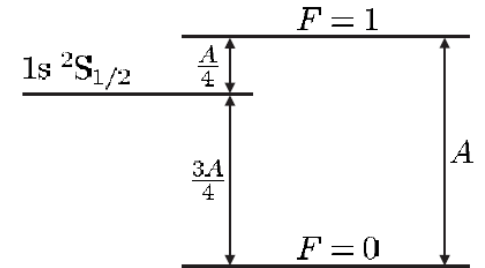
Jądrowy moment magnetyczny $\mu_I = g_I \mu_N \mathbf{I}$ $g_I = 5.6$

$$\mu_N = \mu_B \frac{m_e}{M_p} \simeq \frac{\mu_B}{1836}$$

$$H_{\text{HFS}} = -\mu_I \cdot \mathbf{B}_e$$

$$E_{\text{HFS}} = A \langle \mathbf{I} \cdot \mathbf{J} \rangle = \frac{A}{2} \{F(F+1) - I(I+1) - J(J+1)\} \quad \mathbf{F} = \mathbf{I} + \mathbf{J}$$

$$A = \frac{2}{3} \mu_0 g_s \mu_B g_I \mu_N \frac{Z^3}{\pi a_0^3 n^3}$$



	Fine structure in the LS -coupling scheme	Hyperfine structure in the IJ -coupling scheme
Interaction	$\beta \mathbf{L} \cdot \mathbf{S}$	$A \mathbf{I} \cdot \mathbf{J}$
Total angular momentum	$\mathbf{J} = \mathbf{L} + \mathbf{S}$	$\mathbf{F} = \mathbf{I} + \mathbf{J}$
Eigenstates	$ LSJM_J\rangle$	$ IJFM_F\rangle$
Energy, E	$\frac{\beta}{2} \{J(J+1) - L(L+1) - S(S+1)\}$	$\frac{A}{2} \{F(F+1) - I(I+1) - J(J+1)\}$
Interval rule	$E_J - E_{J-1} = \beta J$ (if $E_{s-o} < E_{re}$)	$E_F - E_{F-1} = AF$ (if $A \gg \Delta E_{\text{Quadrupole}}$)

Efekt Zeemana

$$\begin{aligned}
 H_B &= \frac{\mu_B}{\hbar} (g_S \mathbf{S} + g_L \mathbf{L} + g_I \mathbf{I}) \cdot \mathbf{B} \\
 &= \frac{\mu_B}{\hbar} (g_S S_z + g_L L_z + g_I I_z) B_z
 \end{aligned}$$

$$g_L = 1 - \frac{m_e}{m_{\text{nuc}}}$$

Przesunięcie w energii małe w stosunku do struktury subtelnej

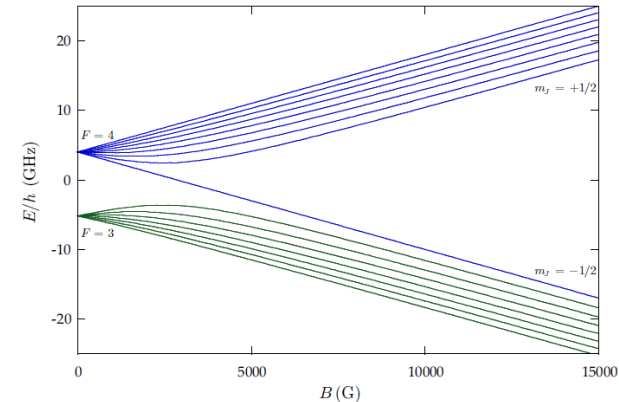
$$H_B = \frac{\mu_B}{\hbar} (g_J J_z + g_I I_z) B_z$$

$$\begin{aligned}
 g_J &= g_L \frac{J(J+1) - S(S+1) + L(L+1)}{2J(J+1)} + g_S \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)} \\
 &\simeq 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)},
 \end{aligned}$$

Przesunięcie w energii małe w stosunku do struktury nadsubtelnej

$$H_B = \mu_B g_F F_z B_z$$

$$\begin{aligned}
 g_F &= g_J \frac{F(F+1) - I(I+1) + J(J+1)}{2F(F+1)} + g_I \frac{F(F+1) + I(I+1) - J(J+1)}{2F(F+1)} \\
 &\simeq g_J \frac{F(F+1) - I(I+1) + J(J+1)}{2F(F+1)}.
 \end{aligned}$$



Termy atomowe

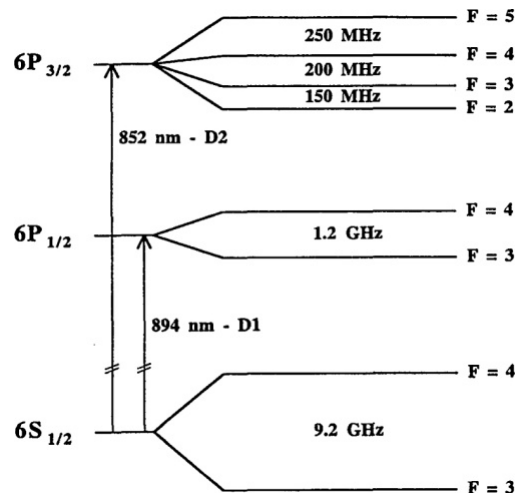
$$2S+1 L_J$$

L – symbol termu

2S+1 – multipletowość

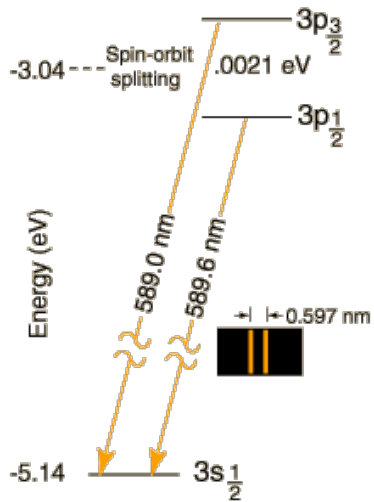
J – wartość sprzężenia LS

L =	0	1	2	3	4	5	6	7	8	9	10	...
	S	P	D	F	G	H	I	K	L	M	N	(dalej alfabetycznie)

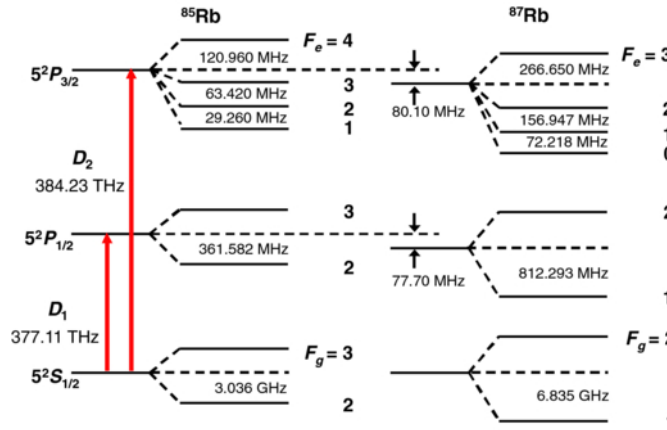


- $l = 0$ "S" states (s for "sharp")
- $l = 1$ "P" states (p for "principal")
- $l = 2$ "D" states (d for "diffuse")
- $l = 3$ "F" states (f for "fundamental")

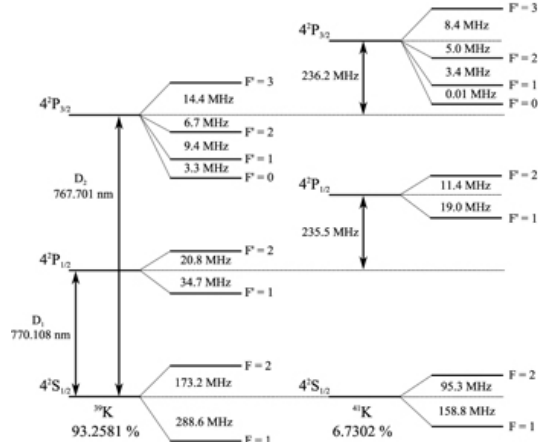
Struktura subtelna i nadsubtelna



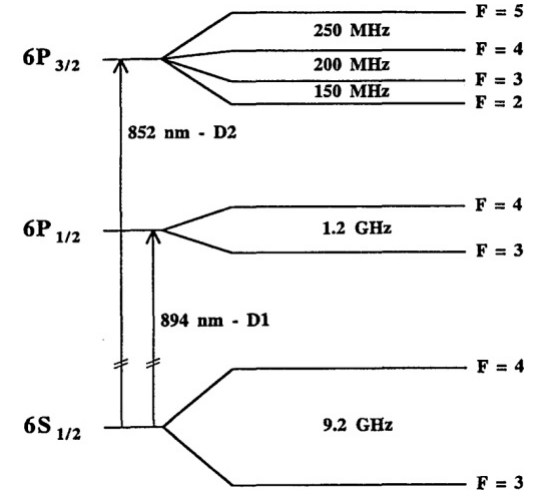
Sód



Rubid



Potas



Cez