

# WYKŁAD 1

## Wstęp do optyki i fizyki materii skondensowanej

Optyka

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Fizyka materii skondensowanej:

Johannes Binder

# Zasady części Optyka

- Wykład przeglądowny
- Ćwiczenia rozszerzające i uzupełniające
- <http://ultracold.fuw.edu.pl/index.php/wdoifms2020/>
- Ocena końcowa: średnia O i FMS

# Literatura

- „Quantum Optics, an Introduction”, M. Lewenstein, A. Sanpera, M. Pospiech  
[www.matthiaspospiech.de/files/studium/skripte/QOscript.pdf](http://www.matthiaspospiech.de/files/studium/skripte/QOscript.pdf)
- „Optyka statystyczna”, Goodman
- „Mechanika kwantowa: teoria nierelatywistyczna”, L.D. Landau, E.M. Lifszic
- „Nonlinear Optics”, Robert W. Boyd
- „Quantum and Atom Optics”, Daniel A. Steck  
<http://atomoptics-nas.uoregon.edu/~dsteck/teaching/quantum-optics/>

# Program części Optyka

1. Pole E-M: przypomnienie podstawowych właściwości, kwantowanie pola, kwantowe stany pola
2. Oddziaływanie atomu z polem elektromagnetycznym
3. Struktura atomu
4. Cząsteczki

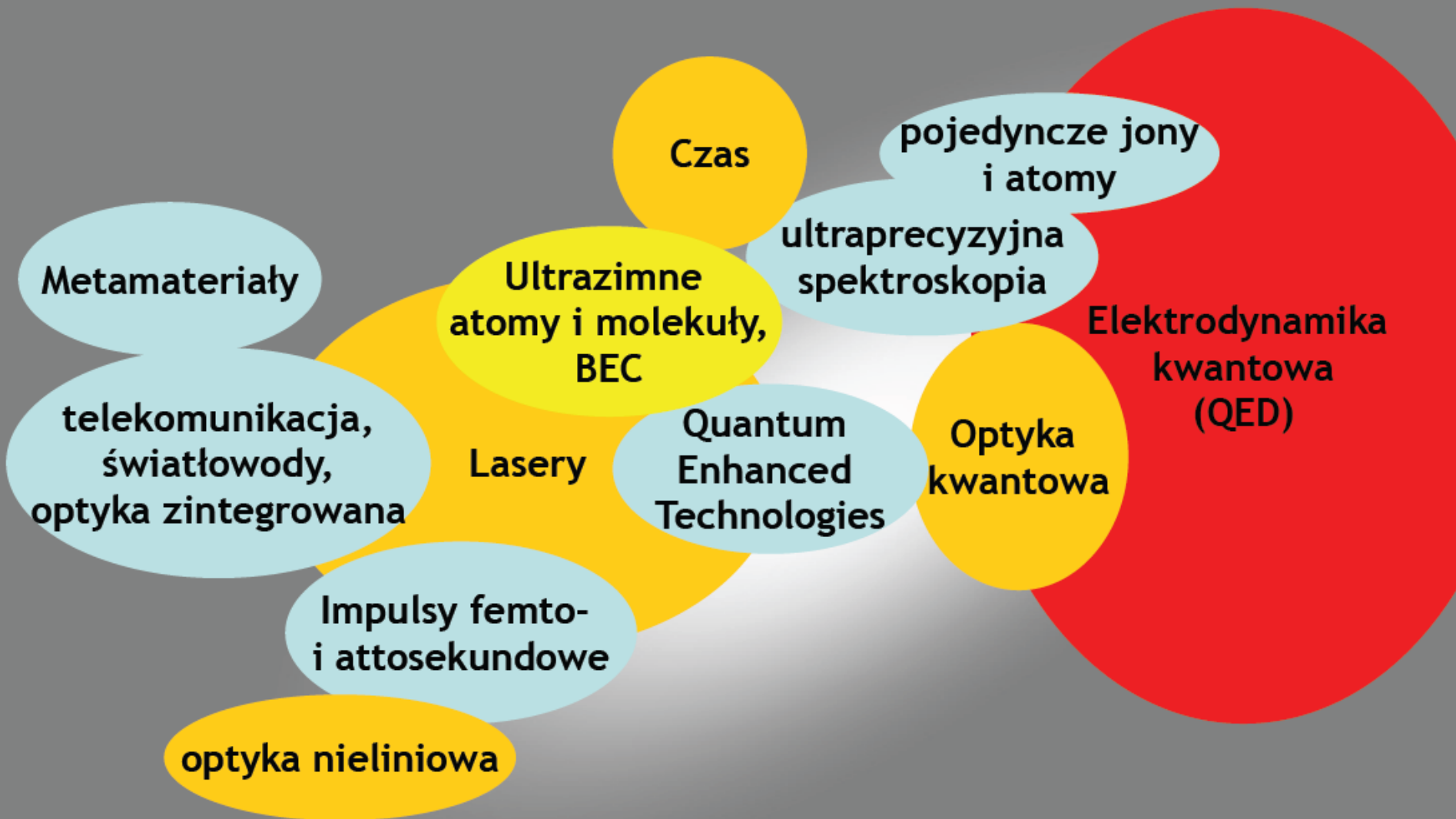
# Zasady części Optyka – Wykł+Ćw

- Prace domowe dopuszczają do egzaminu(>50%)
- Ocena:
  - Egzamin <50% -> sesja poprawkowa
  - Egzamin >50% -> ocena 40% zad. dom. + 60% egzamin

# Zasady części Optyka – tylko wykład

- Co najmniej 50% obecności
- Ocena:
  - 30% lista obecności + 70% zadania domowe
  - możliwy ustny dla zainteresowanych (na 4+ i wyżej)

# Optyka współczesna



# Optyka i Fizyka atomowa na FUW

Inżynieria kwantowych stanów światła ( dr Michał Karpiński),

Obrazowanie przy użyciu światła kwantowego (dr Radek Łapkiewicz),

Pamięci wielomodowe (dr hab. Wojciech Wasilewski, prof. UW),

Femtochemia (dr hab. Piotr Fita),

Lasery femtosekundowe (prof. dr hab. Czesław Radzewicz),

LIDAR i spektroskopia CRDS (prof. dr hab. Tadeusz Stacewicz),

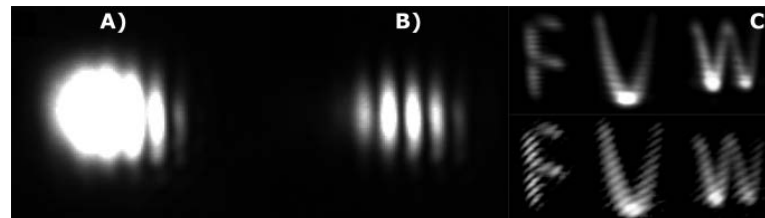
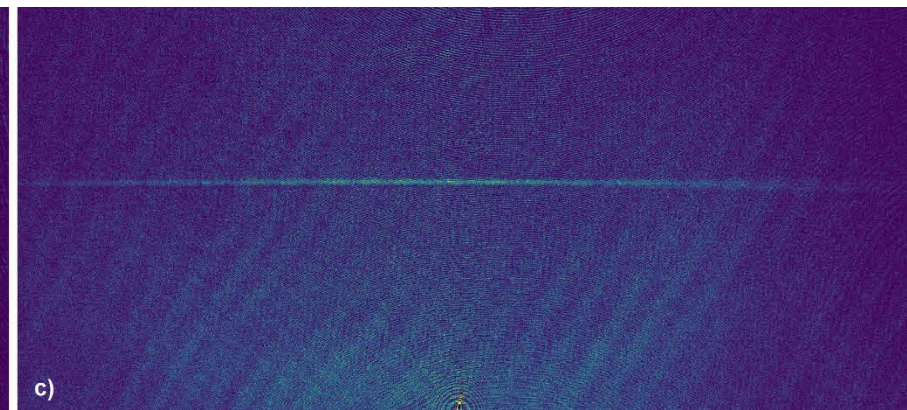
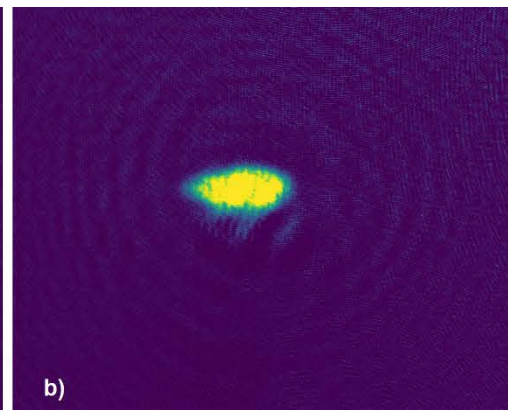
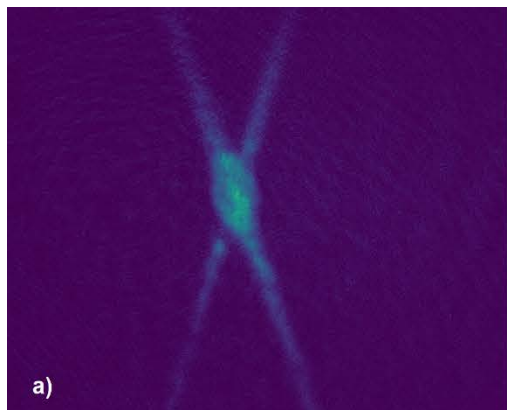
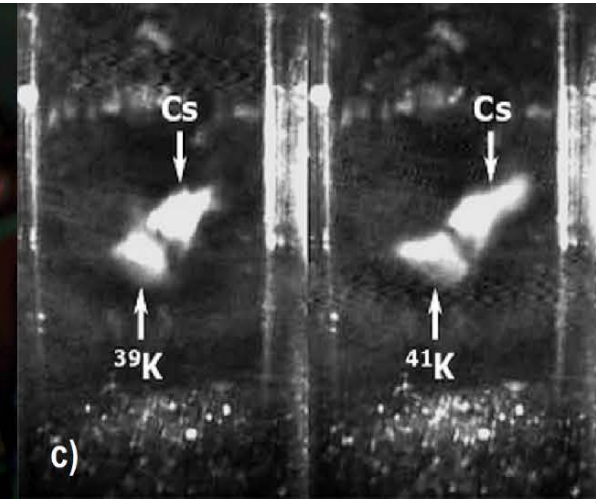
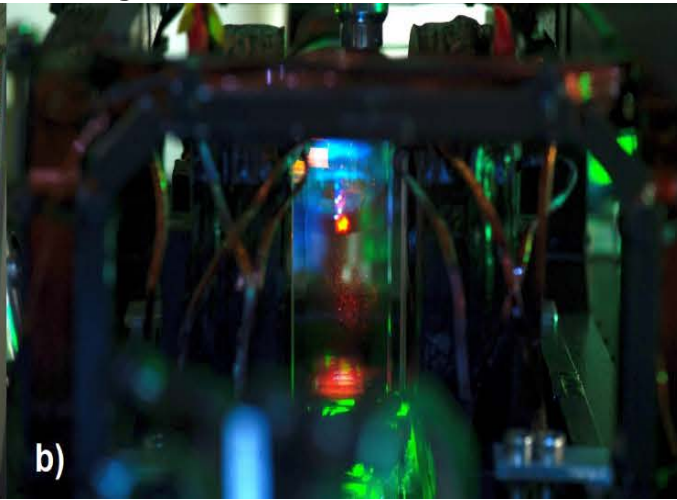
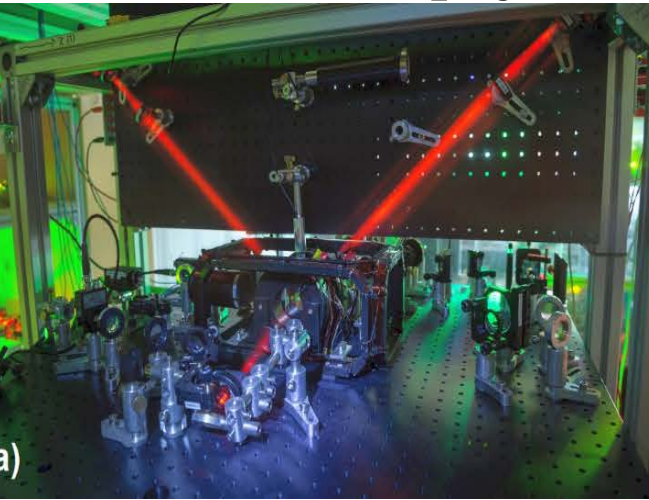
Spektroskopia gorących cząsteczek (prof. dr hab. Paweł Kowalczyk)

Nanostruktury, mikroskopia ramanowska (dr hab. Piotr Wasylczyk)

Ultrazimne gazy kwantowe w sieciach optycznych (dr Mariusz Semczuk)



# Optyka i Fizyka atomowa na FUW



# Klasyczny oscylator harmoniczny

Energia kinetyczna

Energia potencjalna

$$\mathcal{H}(p, q) = \frac{p^2}{2m} + \frac{m\omega^2 q^2}{2}$$

$$\dot{q} = \{q, \mathcal{H}\} = \frac{\partial \mathcal{H}}{\partial p} = p/m$$

$$\dot{p} = \{p, \mathcal{H}\} = -\frac{\partial \mathcal{H}}{\partial q} = -m\omega^2 q$$

Przybliżenie dla większości potencjałów z minimum energii potencjalnej

$$U(q) = U(q_0) + (q - q_0) \left. \frac{dU}{dq} \right|_{q_0} + \frac{(q - q_0)^2}{2!} \left. \frac{d^2U}{dq^2} \right|_{q_0} + \dots$$

$$U(q) = \frac{(q - q_0)^2}{2!} \left. \frac{d^2U}{dq^2} \right|_{q_0} + o(q - q_0)^3 \approx \frac{1}{2} m\omega^2 (q - q_0)^2$$

# Klasyczny oscylator harmoniczny

## Pułapka jonowa

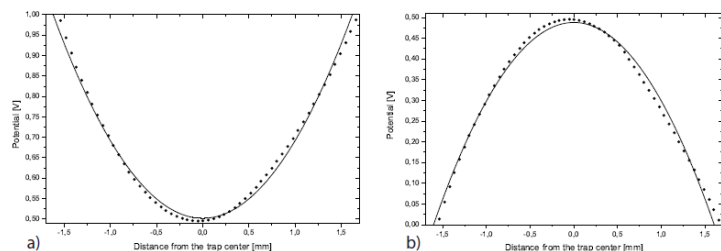


Figure 4.11: Trap potential simulation in the x-y plane - (left) x-direction between RF rods,  $\phi_x[V] = 0.50 \text{ V} + 189886x^2/m^2V$ , (right) y-direction between the grounded rods,  $\phi_y[V] = 0.49 \text{ V} - 189101y^2/m^2V$ . The RF amplitude used in simulations equals 1 V whereas the grounded rods have potential 0 V.

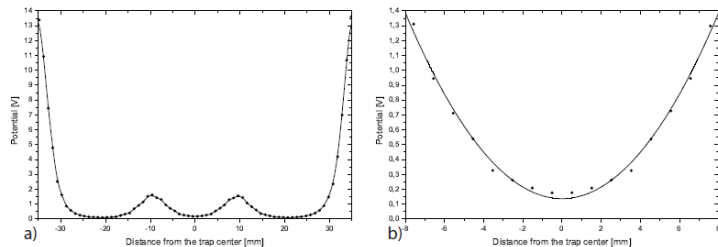


Figure 4.12: Trap potential simulation in the z-direction - (left) view of the axial potential. Peaks correspond to the positions of the rings. The applied voltages are 1000 V for the outer rings and 200 V for the inner rings, (right) magnified central part with the fitted function  $\phi_z = 0.14V + 19540z^2V/m^2$

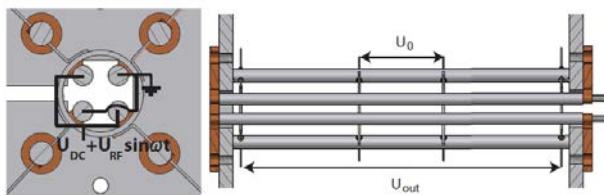


Figure 3.2: Voltage connection - Two opposite rods are connected to a RF signal source, the remaining two are grounded or DC-biased. The rings are DC-biased and provide axial confinement. Depending on the region of interest, ions can be trapped either in the central part or in the entire volume (confined by the voltage  $U_{out}$ ).

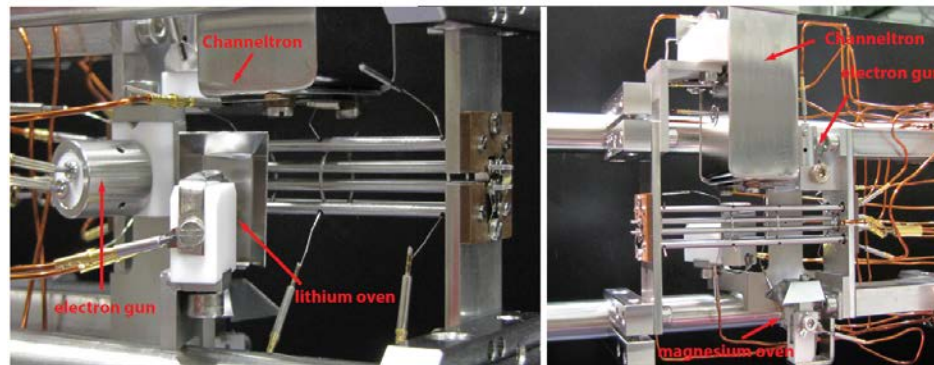


Figure 4.9: Ion trap - Two views of the trap with accompanying equipment: Channeltron (on the top, above the central region), two electron guns (on the top and on the side) as well as Mg and Li ovens (under the trap and on the side, respectively). Ions are created in the outer region to protect the central parts of the electrodes against contamination, which can distort the trapping potential. (left) view from the side of the lithium oven, (right) view from the imaging system side

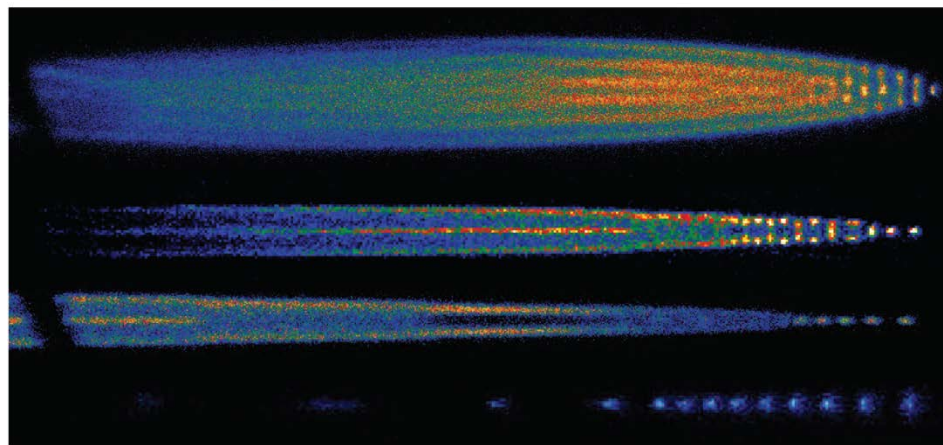
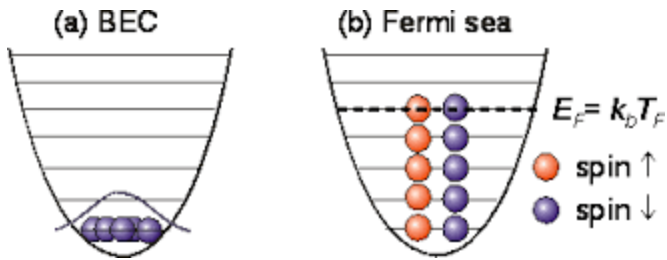


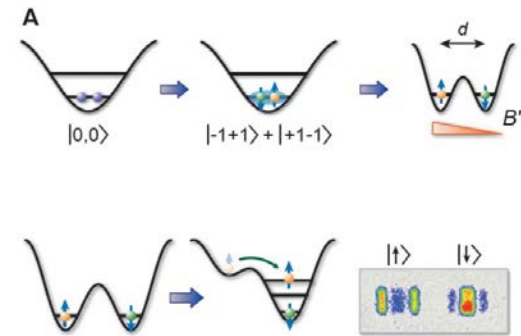
Figure 5.4: Ion crystals -  $^{26}\text{Mg}^+$  crystals for increasing RF amplitude (from the top of the figure). Empty spaces in the string (bottom) are due to  $^{24}\text{Mg}^+$

# Klasyczny oscylator harmoniczny

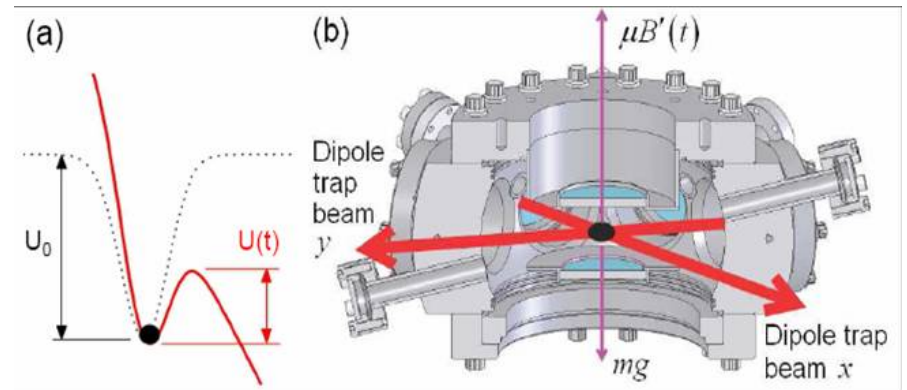
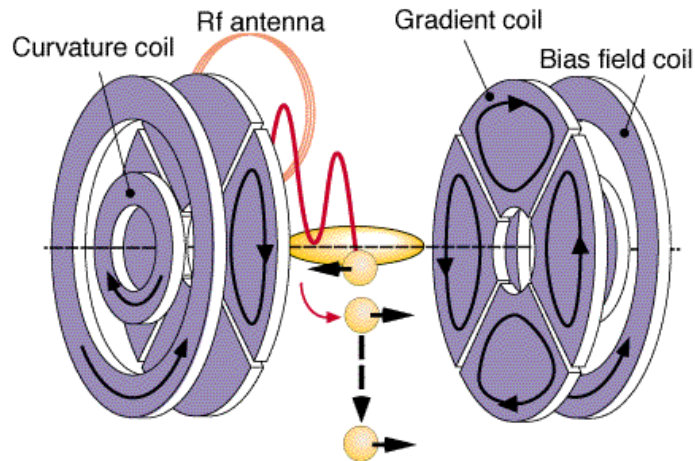
## Pułapka magnetyczna i optyczna



Preparation and detection of Mott states of atoms in a double well potential



## BEC in a "cloverleaf" magnetic trap



# Kwantowy oscylator harmoniczny

$$\mathcal{H}(p, q) = \frac{p^2}{2m} + \frac{m\omega^2 q^2}{2}$$

$$\dot{q} = \{q, \mathcal{H}\} = \frac{\partial \mathcal{H}}{\partial p} = p/m$$

$$\dot{p} = \{p, \mathcal{H}\} = -\frac{\partial \mathcal{H}}{\partial q} = -m\omega^2 q$$

kwantowanie  
kanoniczne

$$q \rightarrow \hat{q}, p \rightarrow \hat{p}$$
$$\{q, p\} \rightarrow [\hat{q}, \hat{p}]/i\hbar$$

$$\hat{\mathcal{H}} = \frac{\hat{p}^2}{2m} + \frac{m\omega^2 \hat{q}^2}{2}$$

$$\frac{d\hat{q}}{dt} = -i[\hat{q}, \hat{\mathcal{H}}]/\hbar = \hat{p}/m$$

$$\frac{d\hat{p}}{dt} = -i[\hat{p}, \hat{\mathcal{H}}]/\hbar = -m\omega^2 \hat{q}$$

$$\hat{\mathcal{H}} = \hbar\omega \left( \hat{a}^\dagger \hat{a} + \frac{1}{2} \right)$$

$$H = \frac{1}{2} \int (\epsilon_0 E^2 + \mu_0 H^2)$$

$$\hat{a} = \sqrt{\frac{1}{2\hbar\omega}} (\omega\hat{q} + i\hat{p})$$

$$\hat{a}^\dagger = \sqrt{\frac{1}{2\hbar\omega}} (\omega\hat{q} - i\hat{p})$$

# Kwantowy oscylator harmoniczny: istotne właściwości

$$[\hat{a}, \hat{a}^\dagger] = 1$$

$$\hat{a}|0\rangle = 0$$

$$\hat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$$

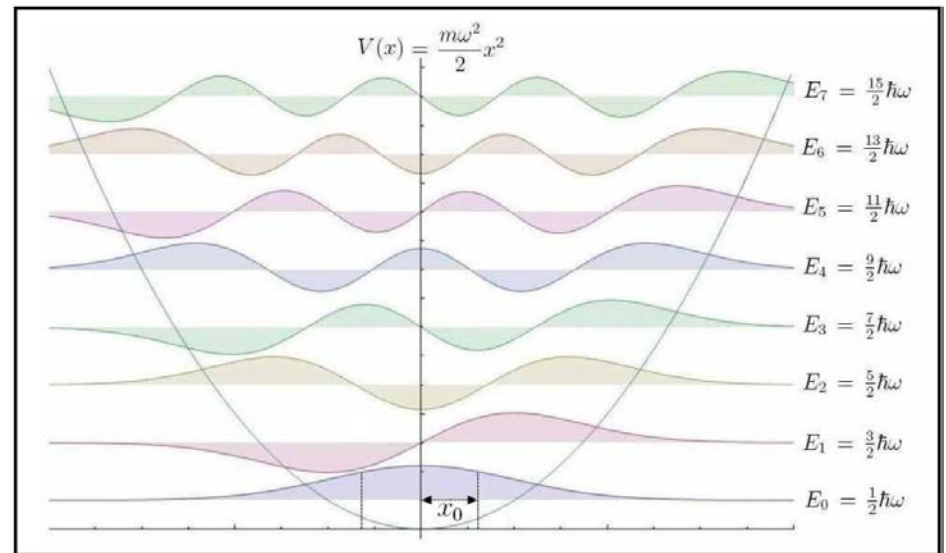
$$\hat{a}|n\rangle = \sqrt{n}|n-1\rangle$$

$$\hat{n}|n\rangle = n|n\rangle$$

$$\hat{a}^\dagger\hat{a} = \hat{n}$$

$$|n\rangle = \frac{(\hat{a}^\dagger)^n}{\sqrt{n!}}|0\rangle$$

$$\sum_{n=0}^{\infty} |n\rangle\langle n| = \hat{I}, \langle m|n\rangle = \delta_{mn}$$



$$\langle n|(\Delta\hat{n})^2|n\rangle = \langle n|\hat{n}^2|n\rangle - \langle n|\hat{n}|n\rangle^2 = 0$$

# Równania Maxwella

$$\nabla \cdot \mathbf{E} = 0$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla \times \nabla \times \mathbf{E} = -\frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

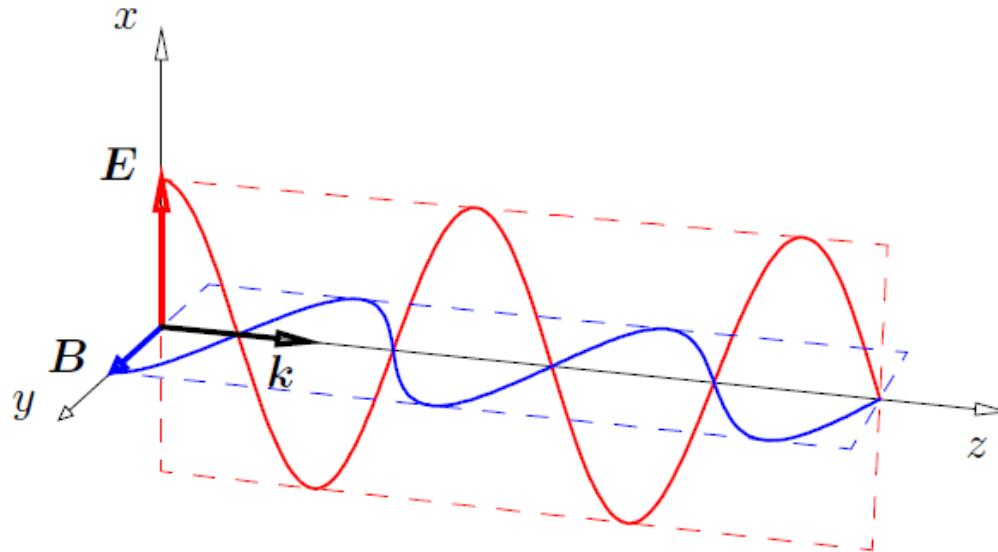
$$\nabla \times (\nabla \times \mathbf{E}) = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E}$$

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

Równanie falowe (Helmholtza)

$$\nabla^2 \mathbf{E} = \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E}_0 f(\mathbf{k} \cdot \mathbf{r} - \omega t)$$



# Równania Maxwella

## fale płaskie

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E}_0 \exp[i(\mathbf{k} \cdot \mathbf{r} - \omega t)]$$

$$\mathbf{B}(\mathbf{r}, t) = \mathbf{B}_0 \exp[i(\mathbf{k} \cdot \mathbf{r} - \omega t)]$$

$$B = \sqrt{\epsilon_0 \mu_0} E$$

$$\mathbf{k} \times \mathbf{B} = -\frac{\omega}{c^2} \mathbf{E} \quad \mathbf{k} \times \mathbf{E} = \omega \mathbf{B}$$

$$\mathbf{k} \cdot \mathbf{B} = 0 \quad \mathbf{k} \cdot \mathbf{E} = 0$$

$\mathbf{E}$ ,  $\mathbf{B}$  i  $\mathbf{k}$  są wzajemnie prostopadłe

Również liniowa kombinacja

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E}^{(+)}(\mathbf{r}, t) + \mathbf{E}^{(-)}(\mathbf{r}, t)$$

$$\mathbf{E}^{(-)} = [\mathbf{E}^{(+)}]^*$$

$$W = 2 \int_V \epsilon_0 \left| \mathbf{E}^{(+)}(\mathbf{r}, t) \right|^2 dV$$

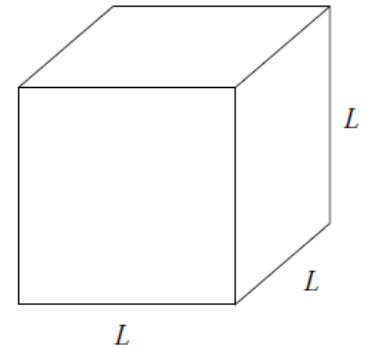


# Pole w rezonatorze: mody (I)

$$E_x(\mathbf{r}, t) = E_x(t) \cos(k_x x) \sin(k_y y) \sin(k_z z)$$

$$E_y(\mathbf{r}, t) = E_y(t) \sin(k_x x) \cos(k_y y) \sin(k_z z)$$

$$E_z(\mathbf{r}, t) = E_z(t) \sin(k_x x) \sin(k_y y) \cos(k_z z)$$

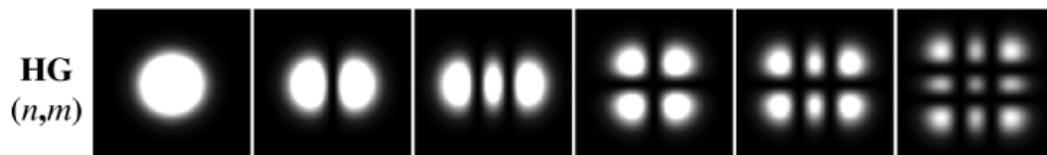
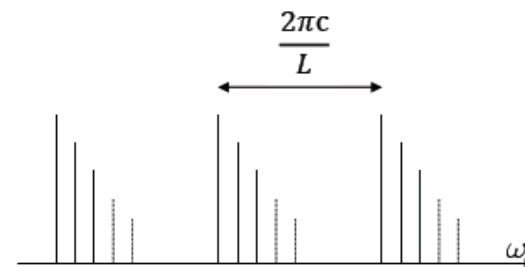
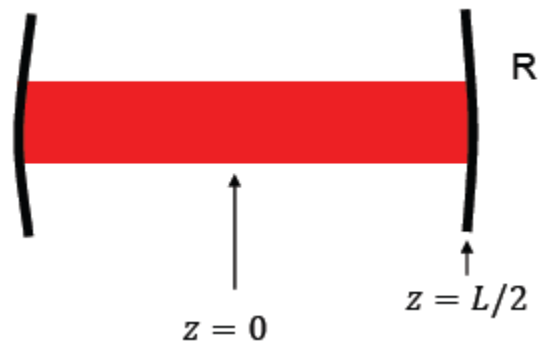


$$k_x = \frac{\pi}{L} n_x, \quad k_y = \frac{\pi}{L} n_y, \quad k_z = \frac{\pi}{L} n_z$$

$$E_x(t) k_x + E_y(t) k_y + E_z(t) k_z = 0$$

$$n_x, n_y, n_z = 0, 1, 2, 3, \dots$$

# Pole w rezonatorze: mody (II)



**HG**  
 $(n,m)$

(0,0)

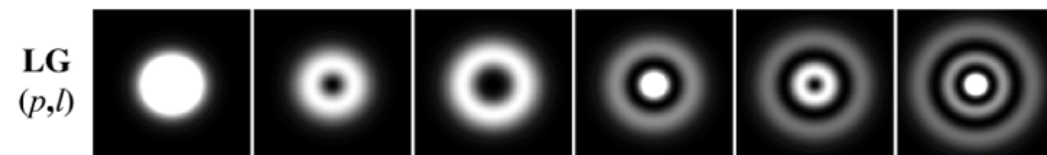
(1,0)

(2,0)

(1,1)

(2,1)

(2,2)



**LG**  
 $(p,l)$

(0,0)

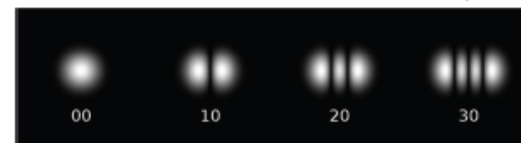
(0,1)

(0,2)

(1,0)

(1,1)

(2,0)



$$u_n(k) = \sqrt{\frac{w_0}{\sqrt{\pi} 2^n n!}} H_n(kw_0) e^{-k^2 w_0^2 / 2}$$

H-G : współrzędne kartezjańskie  
 L-G : współrzędne cylindryczne

# Kwantowanie pola: pole jednomodowe

Punkt wyjścia: rezonator jednowymiarowy o długości  $L$ ,  
pole elektryczne spolaryzowane wzdłuż  $x$ ,  
 $x = 0, y = \frac{L}{2}, k_z = k, \omega = ck$   
fala stojąca  $E_x(z, t) = E_x(t)\sin(kz)$

$$\begin{array}{l}
 E_x(z, t) = \sqrt{\frac{2\omega^2}{\epsilon_0 L}} q(t) \sin kz \\
 B_y(z, t) = \frac{1}{c\omega} \sqrt{\frac{2\omega^2}{\epsilon_0 L}} p(t) \cos kz
 \end{array}
 \left. \vphantom{\begin{array}{l} E_x \\ B_y \end{array}} \right\}
 \begin{array}{l}
 \xrightarrow{\nabla \times E = -\frac{\partial B}{\partial t}} \frac{dp}{dt} = -\omega^2 q \\
 \xrightarrow{\nabla \times B = \frac{1}{c^2} \frac{\partial E}{\partial t}} \frac{dq}{dt} = p
 \end{array}
 \xrightarrow{d/dt} \frac{d^2 q}{dt^2} + \omega^2 q = 0$$

$$\mathcal{H} = \frac{1}{2} (\omega^2 q^2 + p^2)$$

# Kwantowanie pola: pole jednomodowe

Punkt wyjścia: rezonator jednom wymiarowy o długości  $L$ ,  
pole elektryczne spolaryzowane wzdłuż  $x$ ,  
 $x = 0, y = \frac{L}{2}, k_z = k, \omega = ck$   
fala stojąca  $E_x(z, t) = E_x(t)\sin(kz)$

$$\begin{aligned}
 E_x(z, t) &= \sqrt{\frac{2\omega^2}{\epsilon_0 L}} q(t) \sin kz \\
 B_y(z, t) &= \frac{1}{c\omega} \sqrt{\frac{2\omega^2}{\epsilon_0 L}} p(t) \cos kz
 \end{aligned}$$

$$\begin{aligned}
 \nabla \times E &= -\frac{\partial B}{\partial t} \longrightarrow \frac{dp}{dt} = -\omega^2 q \\
 \nabla \times B &= \frac{1}{c^2} \frac{\partial E}{\partial t} \longrightarrow \frac{dq}{dt} = p \xrightarrow{d/dt} \frac{d^2 q}{dt^2} + \omega^2 q = 0
 \end{aligned}$$

$$\mathcal{H} = \frac{1}{2} (\omega^2 q^2 + p^2)$$

$$\begin{aligned}
 \hat{E}_x(z, t) &= \sqrt{\frac{2\omega^2}{\epsilon_0 L}} \hat{q}(t) \sin kz \\
 \hat{B}_y(z, t) &= \frac{1}{c\omega} \sqrt{\frac{2\omega^2}{\epsilon_0 L}} \hat{p}(t) \cos kz \\
 \hat{\mathcal{H}} &= \frac{1}{2} (\omega^2 \hat{q}^2 + \hat{p}^2)
 \end{aligned}$$

# Kwantowanie pola: pole jednomodowe – dygresja(O. H.)

$$\hat{\mathcal{H}} = \hbar\omega \left( \hat{a}^\dagger \hat{a} + \frac{1}{2} \right)$$

$$[\hat{a}, \hat{a}^\dagger] = 1$$

$$\hat{a}|0\rangle = 0$$

$$\hat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$$

$$\hat{a}|n\rangle = \sqrt{n}|n-1\rangle$$

$$\hat{n}|n\rangle = n|n\rangle$$

$$\sum_{n=0}^{\infty} |n\rangle\langle n| = \hat{I}, \quad \langle m|n\rangle = \delta_{mn}$$

$$\hat{a}^\dagger \hat{a} = \hat{n}$$

$$\hat{a} = \sqrt{\frac{1}{2\hbar\omega}} (\omega\hat{q} + i\hat{p})$$

$$\hat{a}^\dagger = \sqrt{\frac{1}{2\hbar\omega}} (\omega\hat{q} - i\hat{p})$$

$$|n\rangle = \frac{(\hat{a}^\dagger)^n}{\sqrt{n!}} |0\rangle$$

$$\langle n|(\Delta\hat{n})^2|n\rangle = \langle n|\hat{n}^2|n\rangle - \langle n|\hat{n}|n\rangle^2 = 0$$

# Kwantowanie pola: pole jednomodowe

$$\hat{\mathcal{H}} = \frac{1}{2}(\omega^2 \hat{q}^2 + \hat{p}^2)$$

$$\begin{array}{l} \hat{a} = \sqrt{\frac{1}{2\hbar\omega}} (\omega\hat{q} + i\hat{p}) \\ \hat{a}^\dagger = \sqrt{\frac{1}{2\hbar\omega}} (\omega\hat{q} - i\hat{p}) \end{array} \quad \downarrow \quad \begin{array}{l} \hat{q} = \sqrt{\frac{\hbar}{2\omega}} (\hat{a} + \hat{a}^\dagger) \\ \hat{p} = -i\sqrt{\frac{\hbar\omega}{2}} (\hat{a} - \hat{a}^\dagger) \end{array}$$

$$\hat{\mathcal{H}} = \hbar\omega \left( \hat{a}^\dagger \hat{a} + \frac{1}{2} \right)$$

$$\hat{E}_x(z, t) = \sqrt{\frac{\hbar\omega}{\epsilon_0 L}} (\hat{a} + \hat{a}^\dagger) \sin kz,$$

$$\hat{B}_y(z, t) = -i \frac{1}{c} \sqrt{\frac{\hbar\omega}{\epsilon_0 L}} (\hat{a} - \hat{a}^\dagger) \cos kz$$

$$\hat{a}^\dagger(t) = \hat{a}^\dagger(0) \exp(i\omega t)$$

$$\hat{a}(t) = \hat{a}(0) \exp(-i\omega t)$$

$$\longleftarrow \frac{d}{dt} \hat{a} = \frac{i}{\hbar} [\hat{\mathcal{H}}, \hat{a}]$$

# Kwantowanie pola: pole wielomodowe

$$\hat{\mathcal{H}} = \sum_k \hbar\omega_k (\hat{a}_k^\dagger \hat{a}_k + 1/2)$$

$$\begin{aligned} \hat{q}_k &= \sqrt{\frac{\hbar}{2\omega}} [\hat{a}_k^\dagger + \hat{a}_k] & \hat{a}_k &= \frac{1}{\sqrt{2\hbar\omega}} [\omega_k \hat{q}_k + i\hat{p}_k] \\ \hat{p}_k &= i\sqrt{\frac{\hbar\omega}{2}} [\hat{a}_k^\dagger - \hat{a}_k] & \hat{a}_k^\dagger &= \frac{1}{\sqrt{2\hbar\omega}} [\omega_k \hat{q}_k - i\hat{p}_k] \end{aligned}$$

$$\hat{\mathcal{H}} = \sum_k \hbar\omega_k \hat{a}_k^\dagger \hat{a}_k + \text{“}\infty\text{”}$$