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Repumping laser for laser cooling of caesium

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Summary

This work presents all the steps leading to the construction of a semiconductor laser and frequency tuning of light that it emits. In the described system, the constructed laser is injection locked to an external cavity diode laser (ECDL). This approach allows for obtaining much cheaper laser with the same spectral properties as the ECDL. Once it is seeded and injection locked, such a laser can be handled independently. The work includes experimental data and their analysis in terms of properties influencing the injection locking. It also contains a review of a method which allows for further detuning of the constructed laser light frequency so that it can be used as a repumping beam in the laser cooling of caesium atoms.

Key words

atomic physics, injection locking, laser cooling

Area of study (codes according to Erasmus Subject Area Codes List)

13.2 Physics

The title of the thesis in Polish

Laser repompujący do chłodzenia laserowego cezu

Contents

The aim

Laser cooling of alkali atoms requires two frequency components (so called cooling and repumping frequencies) to address the atoms occupying both hyperfine levels of the ground state. For heavy atoms, like caesium, typically two separate frequency stabilized lasers are used for this purpose. Given that for caesium the required intensity of light generated at the repumping frequency is one - two orders of magnitude lower than needed at the cooling frequency, it is desirable to develop a technique for generating the repumping light in a cost-effective way. Current approach, employed in the Laboratory of Ultracold Quantum Gases led by dr. Mariusz Semczuk, uses an expensive (>30 000 EUR) commercial MOPA system[1] TA PRO from Toptica (tapered amplifier seeded by an external cavity diode laser) and it is desirable to "free" this laser for other applications.

This thesis presents a construction of semiconductor laser system which can be used as a source of repumping light for laser cooling of caesium atoms. The constructed laser (called here "slave laser") is injection locked to an external cavity diode laser (ECDL, called "master laser"). Injection locking is obtained by seeding the laser cavity of a free running laser diode with the light from a reference laser. The seeding beam is injected into the slave's cavity through an optical isolator equipped with beam splitting polarizers. The long term goal of the work reported in this thesis is to develop a know-how that can be later used to construct other laser systems that will operate at frequencies away from the frequency stabilized lasers available in the Laboratory of Ultracold Quantum Gases. As such, the technique described in this work could be implemented to generate sources for imaging of ultracold atoms at high magnetic fields, for creation of deep optical lattices or for coherent population transfer, to name a few.

After the introductory Chapter 1, the theoretical introduction to the laser cooling of caesium, laser diode operation and the injection locking concept are presented in Chapter 2. The description of the steps leading to the construction of the semiconductor laser and the optical system allowing for its seeding with the ECDL light are detailed in Chapter 3. It contains information about required elements, their function in the system and their installation.

Chapter 4 contains the measurements and analysis of experimental data. The results are accompanied by a short description of each measurement method relevant to the understanding of a given approach. This part contains proofs derived from experimental data that the injection locking in the system has been successful and it gives an overview of optimal parameters for this phenomenon to be observed. The concluding Chapter 5 summarizes the work and discusses steps that are necessary to turn the constructed laser system into a repumping laser.

[1] Master Oscillator Power Amplifier

1 Introduction

1.1 Injection locking in atomic physics

The first successful experiment that confirmed injection locking theory [1] was conducted in 1966 by Stover and Steier [2]. The scientists measured a beat note signal between two lasers, where the light emitted by one of them ("master laser") was injected into the cavity of the second laser ("slave laser"). In that work they used two helium-neon lasers, each having a cavity length controlled by a piezoelectric element. By scanning the frequency of the seeding light they showed that the frequency of the slave, when injection locked, followed the changing frequency of the master laser for a certain range of frequency differences between lasers. They also showed that for higher intensity of the injected master light the injection locking is possible for larger range of frequency differences between the lasers.

In 1994 [3], Linlin Li presented analytical formulas which explained semiconductor laser's behaviour with external light injection. His description includes performance of the laser biased near the threshold, both below and above. It also deals with the theory standing behind the gain change and the threshold change of the semiconductor laser when external light is introduced into its cavity. He was the first one to include spontaneous emission effects in his calculations and predicted that the threshold would decrease with increase of injection power. The theory of Linlin Li was confirmed experimentally for the first time in 1998 by S. Sivapraskasam and Ranjit Singh [4].

The techniques used in atomic physics, such as Raman spectroscopy, laser cooling, optical pumping, electromagnetically induced transparency or Raman sideband cooling require laser light at various frequencies, often separated by the hyperfine splitting of the ground state of the atoms used in a given experiment. For atoms such as rubidium and caesium, this frequency difference is in the range of several GHz. Injection locking might then become an useful technique that enables phase locking between the master and the slave while creating such a frequency detuning.

In 1996 the frequency modulation with acusto-optical modulators (AOM) together with injection locking technique were used to detune two light beams frequencies by 9.2 GHz. A broadband AOM was used to perform injection locking of two diode lasers to the +1 and -1 order diffracted beam resulting, correspondingly, in +4.6 GHz and -4.6 GHz detuning from the master central frequency [5]. The frequency shifted light already had the frequencies required for experiments and the two lasers inherited their spectral properties. Such a solution is limited by the low diffraction efficiency of AOMs at high frequencies.

In 1997 a novel technique was presented which also showed that injection seeding can be used for optical phase locking between two diode lasers detuned from each other by several GHz [6]. The researchers from Strathclyde University modulated laser diode current with RF signal which resulted in a creation of sidebands in the laser light spectrum. Their method used the generated sideband to make it injection locked to the master's frequency. Laser systems which use injection locking do not require wideband electronic phased-locked loops and the technique described in this paragraph does not require any optical elements for the frequency modulation of master's light. This makes the whole system less expensive and less complicated. It was also proposed that the system using sideband injection locking could be used in laser cooling or Raman spectroscopy. The team of P. L. Gould used this method in 2001 [7] to create a laser system that could be used for laser cooling of ^{85}Rb and ^{87}Rb with the detuning of the lasers tunable within 2.4-3.7 GHz and 6.5-8.6 GHz ranges.

The main inspiration for this work is the setup of Junmin Wang et al. from 2011 which presents a laser system for a caesium magneto-optical trap (MOT) [8] which requires laser beams of two different frequencies called cooling and repumping. For caesium, these two frequencies differ by about 9.2 GHz. Such a detuning is relatively high and the frequency modulation on such a scale becomes a technical challenge. Their system uses the technique described above, that is injection locking of the sideband generated by the laser diode current modulation. This approach enables easy frequency tuning of the seeded laser by changing the modulation frequency of the current seeding the laser diode.

Similar approach can be used for tapered amplifiers seeded with master laser light. Tapered amplifier current modulation results in generation of sidebands in its spectrum which have lower intensity than the carrier wave intensity. As presented in [9], the sideband's intensity might be high enough to use the amplifier's output light directly for the laser cooling since it contains light of both cooling and repumping frequencies. It was reported that the method allowed for creating magneto-optical trap of ^{87}Rb atoms, for which the cooling and repumping frequency detuning is equal to 6.6 GHz.

Injection locking is also a useful and low-cost tool for increasing intensity of desired frequency light in the experimental system. In ref. [10] the injection locking of a low-cost high-power laser diode is presented. The laser diode used in this experiment can produce up to 500 mW of multimode light. It was shown that injecting 10 mW of monomode master laser light enabled tuning of at least 50% of the light generated by the slave with the 150 MHz of spectrum bandwidth effectively cleared from noise due to laser diode multimode operation. It was proposed that such a system could be used in high-resolution spectroscopy. It was also successfully used as a part of a strontium atoms laser cooling system.

Injection locking is often used in atomic physics experiments, especially in applications requiring phase coherence of frequency components in applications such as Raman sideband cooling or electromagnetically induced transparency.

2 Theoretical introduction

2.1 Laser cooling of caesium

Laser cooling allows for decreasing the mean kinetic energy of atoms. It requires light at frequency nearly resonant with an atomic transition and spatially varying magnetic field (e.g. of a quadrupole type). Directing such a nearly resonant laser beam at the cloud of atoms leads to acts of absorption and emission of photons leading to transfer of momentum and radiative force acting on the atoms. Each photon carries momentum that is equal to

$$\bar{p} = \hbar\bar{k} \tag{1}$$

where $|\bar{k}| = \frac{2\pi}{\lambda}$ is the wavenumber, which is inversely proportional to the wavelength, $\hbar$ is the Plancks' constant divided by 2π. The photon's energy is given by the formula:

$$E = \hbar\omega \tag{2}$$

where ω is its angular frequency. An atom moving towards a laser beam can absorb a photon from the laser and this will decrease its kinetic energy. What is crucial in laser cooling is that absorption is followed by spontaneous emission of a photon which is isotropic, so after many acts of absorption and emission the net momentum transfer due to emission averages out to zero. This way the momentum of each individual atom and mean kinetic energy of atom gas can be significantly decreased.

Figure 1 presents the hyperfine structure of the D2 line in caesium. The allowed transitions due to the total angular momentum, F, are these for which ΔF = 0, $\pm$1. Laser cooling of caesium requires two laser beams called cooling and repumping. The cooling beam is chosen in such a way that the excited atoms relax to the same state from which they were excited. For D2 line in caesium atoms, the cooling beam is chosen to be resonant with transition from F=4 ground state to F'=5 excited state. According to transition rules, atoms in excited state F'=5 can relax only to the F=4 ground state. However, due to relatively small splitting of energy levels of the $6P_{3/2}$ state, the cooling beam can cause off-resonant transitions from F=4 to F'=4. What follows, atom can relax into the F=3 ground level. It causes depopulation of F=4 ground state level. The repumping light beam, resonant with F=3 to F'=4 transition, is introduced in the system. The role of repumping beam is to remove atoms from ground level F=3 so that they can take part in the cooling cycle again.

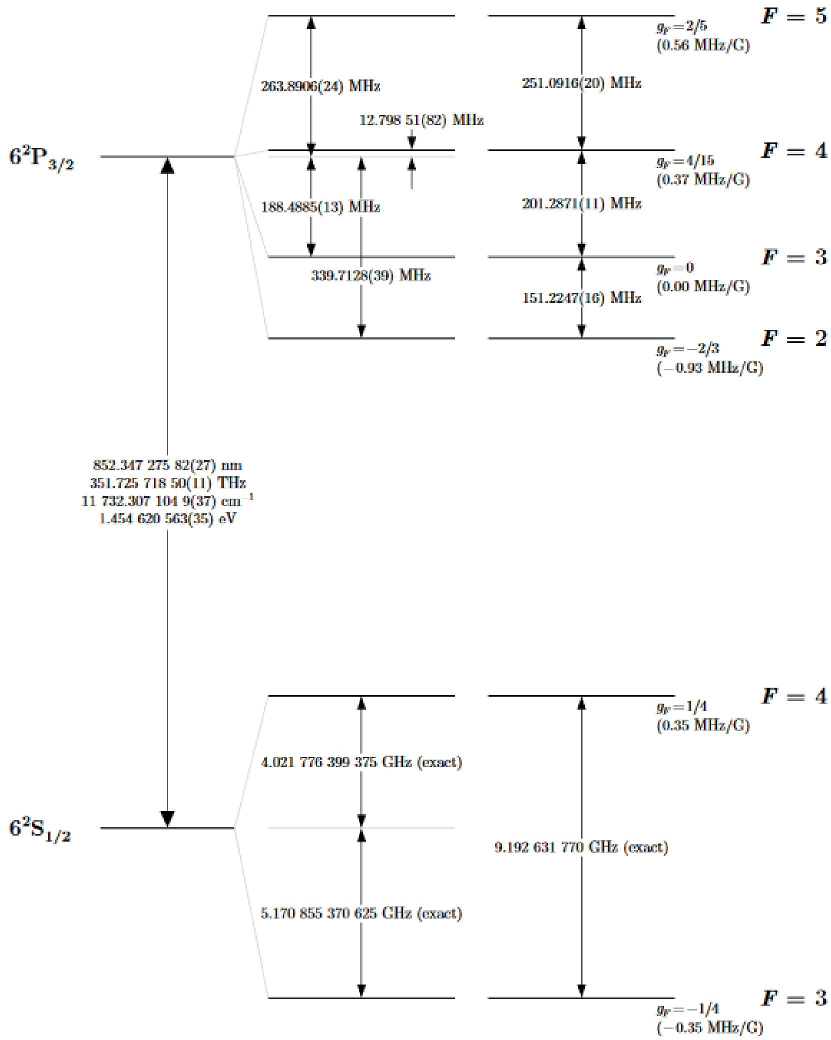

Figure 1: D2 line of hyperfine structure of caesium. In the laser cooling process, the cooling beam is resonant with transition from the F=4 ground state to the F'=5 excited state and the repumping beam is resonant with transition from the F=3 ground state to the F'=4 excited state. The cooling and repumping frequencies are detuned from each other by about 9.2 GHz. Figure taken from [11].

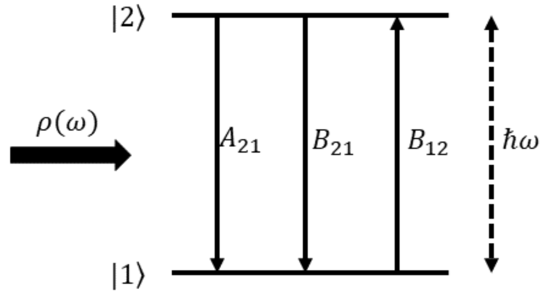

Figure 2: Schematical representation of light-matter interactions in two level atomic system, where $\rho(\omega)$ is the electric field power density, $\hbar\omega$ is photon energy resonant with the transition, A_{21}, B_{21} and B_{12} are the Einstein coefficients for spontaneous emission, stimulated emission and absorption.

2.2 Semiconductor laser

The easiest way to study properties of laser-atom interaction is by considering a two energy level atomic system. There are three possible events occuring in such a system interacting with the light: absorption and stimulated emission, which are dependent on the light density in the medium and spontaneous emission, which is not dependent on this factor. These events are schematically presented in Figure 2 and labelled with the Einstein coefficients, where B_{12} is related to the absorption, A_{21} is related to the spontaneous emission and B_{21} is related to the stimulated emission process. The Einstein coefficients' unit is $1/s$ and they are used to describe the rate equations for the ground state:

$$\frac{dN_1}{dt} = A_{21}N_2 + B_{21}N_2\rho(\omega) - B_{12}N_1\rho(\omega) \tag{3}$$

and for the excited state:

$$\frac{dN_2}{dt} = -A_{21}N_2 - B_{21}N_2\rho(\omega) + B_{12}N_1\rho(\omega) \tag{4}$$

where $\rho(\omega)$ is the energy intensity of the magnetic field. For the laser action to occur, the population inversion is required. The following equation must be satisfied:

$$\frac{N_2}{N_1} = \frac{B_{12}N_1\rho(\omega)}{A_{21} + B_{21}N_1\rho(\omega)} > 1 \tag{5}$$

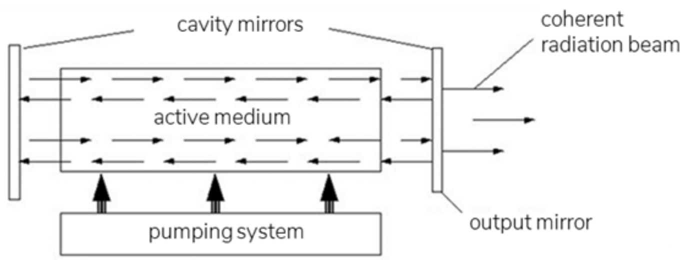

Figure 3: Laser diode structure scheme. Active medium is enclosed between two mirrors of different reflectivity. The emitted light propagates in the optical cavity and is transmitted at its one end. Emitted light beam is coherent. Figure is adapted from [13].

In the semiconductor laser, the population inversion is obtained through electrical pumping. The semiconductor crystal with a p-n junction, which is the laser diode's active medium, is enclosed between two reflective surfaces (Figure 3). The electrical current pumps electrons from the valence to the conduction band. During the process of recombination of an electron in the conduction band with a hole in the valence band, a photon is emitted. If the injected electrical power is high enough, the population inversion can be obtained due to non-zero lifetime of the electron in the excited state; it takes more time for the recombination process to occur than for the excitation process to occur. Emitted photons propagate inside the laser diode's cavity causing stimuated emission. Photons emitted into the cavity through stimulated emission are coherent which is an important characteristic of the laser light. Since the reflectivity of one mirror is lower than of the other one, generated photons have a way to escape the optical cavity.

There are several factors influencing the spectral properties of light emitted by the laser diode. One of them is the energy gap of the semiconductor crystal which depends on the crystal's structure. The energy gap determines the energy of emitted photons and it can be manipulated to some degree with the laser diode temperature or current intensity. Another important factor is the length of the cavity of a laser diode. It determines which longitudinal light modes are enhanced. Since the cavity enhances propagation of descrete wavelengths, the mode hopping is a common phenomenon occuring in the laser diode. It is the reason why the frequency change is not linear for the temperature or current intensity change. Furthermore, the laser diode's cavity length can also change with temperature. In the laser diodes based on AlGaAs crystal, the cavity length increases 0.06 nm/K while the central frequency of generated light due to temperature induced energy gap change increases 0.25 nm/K [12].

2.3 Injection locking

Injection locking allows for obtaining the same spectral characteristics of the light emitted by a slave laser as those of the master laser's light. In other words, it enables frequency tuning of the slave light and, to some extent, control over the its spectral width. The experimental procedure requires sending some optical power of master light beam into the slave's laser diode cavity (Figure 4). The central frequency of the free-running slave laser should be close to the master's central frequency.

Signal gain in laser diode cavity can be described as follows:

$$\tilde{g}(\omega) = \frac{1-R}{1-G_{rt}(\omega)} = \frac{1-R}{1-G(\omega)cos[\phi(\omega)] + iG(\omega)sin[\phi(\omega)]} \tag{6}$$

where R is the round trip decrease of free-running laser oscillations due to injection locking, $G_{rt}(\omega) = G(\omega)e^{-i\phi(\omega)}$ and $G(\omega)$ is equal to round trip gain magnitude that accounts for internal losses, finite mirror reflectivities and laser gain and $\phi(\omega)$ is a round trip phase shift. In the limit of the round trip gain magnitude $G(\omega) \to 1$, signal gain close to the ω_0 mode is now:

$$\tilde{g}(\omega) = \frac{1-R}{1-G+iGT(\omega-\omega_0)} \tag{7}$$

and also the round trip decrease of signal due to injection locking can be approximated as follows if $R \to 1$:

$$R = e^{-\gamma_e T} \approx 1 - \gamma_e T \tag{8}$$

where γ_e is the external decay rate and T is the cavity round trip period. Using these two formulas and knowing that G factor value is close to 1, the injected signal gain dependency can be approximated by:

$$|\tilde{g}(\omega)|^2 = \frac{\gamma_e^2}{(\omega-\omega_0)^2} \tag{9}$$

The assumption that the output power of the laser that is injection locked cannot be higher than the power of the free-running laser leads to the formula:

$$|\tilde{g}(\omega)|^2 I_1 = \frac{\gamma_e^2}{(\omega-\omega_0)^2} I_1 \approx I_0 \tag{10}$$

where I_1 and I_0 are the intensities of the injected light and the light emitted by free-running laser. Hence, the frequency detuning for which the injection locking is possible is determined by:

$$|\omega_1 - \omega_0| \approx \gamma_e \frac{E_1}{E_0} \approx \frac{\omega_0}{Q_e}\sqrt{\frac{I_1}{I_0}} \tag{11}$$

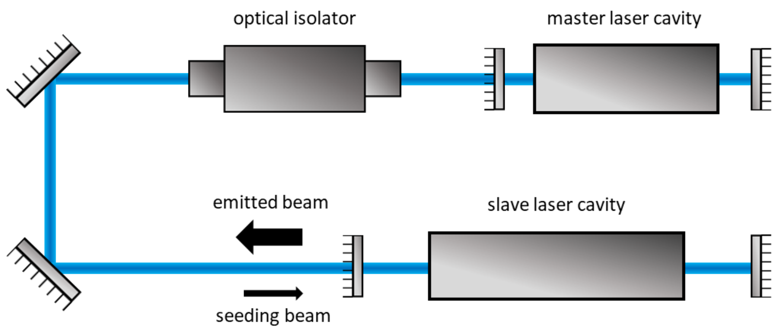

Figure 4: Basic optical scheme for injection locking. The light from the master laser, the seeding beam, is sent into the slave laser cavity. The optical isolator prevents light emitted by the slave from entering master's cavity.

where E_1 and E_0 represent electromagnetic field of injected light and of free-running laser respectively, and Q_e is a quality factor of the cavity. The resulting injection window is equal to:

$$\Delta\omega_{lock} \approx 2\gamma_e \frac{E_1}{E_0} = 2\frac{\omega_0}{Q_e}\sqrt{\frac{I_1}{I_0}}. \tag{12}$$

The above equation shows that it is easier to meet the injection locking requirements when the frequencies of free-runing slave and master light do not differ much and that the injection locking windows are wider when higher powers of injected master light are used.

3 The slave laser setup

There are several parts that can be found in typical semiconductor laser, most of which are shown in Figure 5. The most important part is the laser diode which emits light due to electrical pumping provided by an external power supply. The laser diode itself is very sensitive to the parameters of the driving current and temperature of the environment. These dependencies for common AlGaAs lasers (like the one used in this thesis) are around 1 GHz/mA and +0.06 nm/K (within one longitudinal cavity mode) [12]. Their primary source is the bandgap energy dependence on temperature and a band-filling effect caused by the current injected into the active layer as well as refractive index change of the active layer of the semiconductor induced by the Joule heating associated with injected drive current density. To assure stable frequency of emitted light, the current source must be extremely stable which is provided by a PID controller (proportional-integral-derivative controller). It follows that the current control is not enough for stable laser diode work and another required part is a temperature controller. This controller reads an information about the laser diode's temperature, usually from an electronic temperature sensor, to generate proper feedback and send it to some cooling element. A heat sink is required for better cooling efficiency, and it is also important that the heat is well transferred from the element to the laser diode. This is why most of the mountings are made of metals, very often aluminium, because they guarantee both the mechanical stability and good heat transfer away from the diode. Once the diode work is stable, another important factor to look at is how the beam of emitted light propagates. In most cases the beam generated by the laser diode is diverging so a short focal length lens is used to collimate the beam. After collimator there is an optical isolator which prevents light reflected off elements in optical setup from coming back to the laser diode. It is a device which uses polarizers and a Faraday rotator to pass light in forward direction and block it in the reverse one. All of these elements are put inside an enclosure, typically metal one. It makes the laser an independent part which can be easily moved and installed in a desired place. Its aim is also to ensure stable conditions for the elements inside and to protect them from the dust. When it comes to protection, there is one more element worth mentioning and it is a laser diode protecting circuit. It is soldered close to the laser diode's terminals and it prevents it from damage due to incorrect use or sudden voltage changes. The later part of this section is an overview of the elements used for the construction of the laser.

3.1 Laser diode and the collimator

For this project, a commercially available Fabry-Perot cavity laser diode was chosen, model L852P100 from Thorlabs. The data provided by the manufacturer shows that this laser diode has a central wavelength of about 852 nm, its output power can reach up to 100 mW and that it has a single spatial mode. In our setup the laser diode is mounted inside a commercial collimator housing with an integrated aspheric lens. The collimator assures that the laser diode

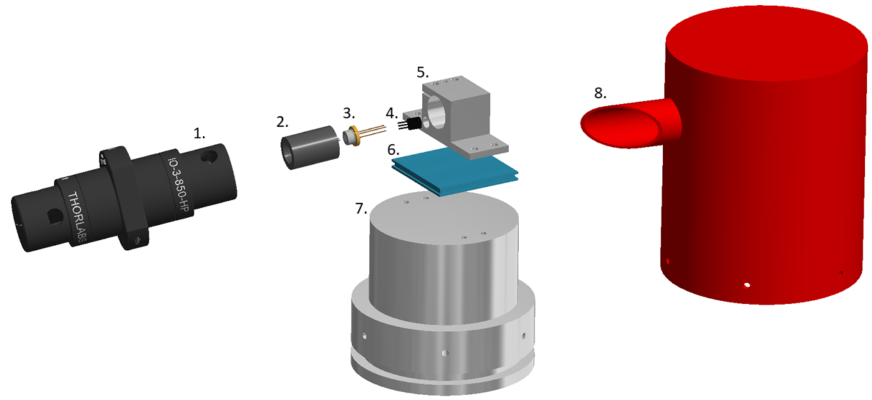

Figure 5: Elements of the semiconductor laser: 1) optical isolator, 2) collimation tube, 3) laser diode, 4) electronic temperature sensor, 5) collimator holder, 6) cooling element (Peltier module), 7) base, 8) cover.

and the focusing lens are mounted on the same axis (Figure 6). By adjusting the distance between the light source and the lens, the emitted beam can be collimated, i.e. shaped in such a way that its size does not change significantly after free space propagation. Collimation tube used here allows for easy adjustment of lens position from outside of the tube. In the constructed laser, collimation was optimized by comparing the beam size at locations separated by 3 meters.

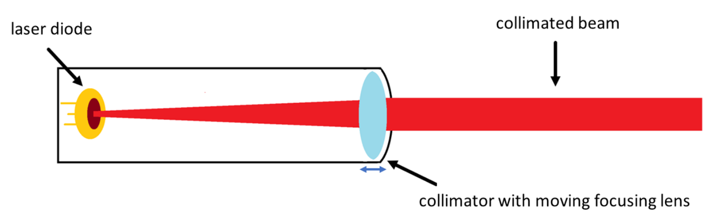

Figure 6: Laser diode diverging beam collimation with focusing lens.

3.2 Laser diode protection circuit

The laser diode lifetime depends on how it is used. The manufacturer provides data about current and temperature ranges which are safe for the diode.

When those parameters are out of range, a damage may occur. The protection circuit prevents from laser diode damage due to:

- high frequency currents by filtering them out,
- electrical surges occuring in the power lines,
- user mistakes such as applying reverse current; it is prevented by placing a diode with reverse direction of conduction in parallel to the laser diode,
- electrostatic discharge of accumulated charges, which can be avoided by placing a capacitor in the circuit.

The scheme of laser diode protection circuit is presented in Figure 7 and it was taken from [19]. The printed circuit board (PCB) was designed with EasyEDA software and made using method of photoresist transfer.

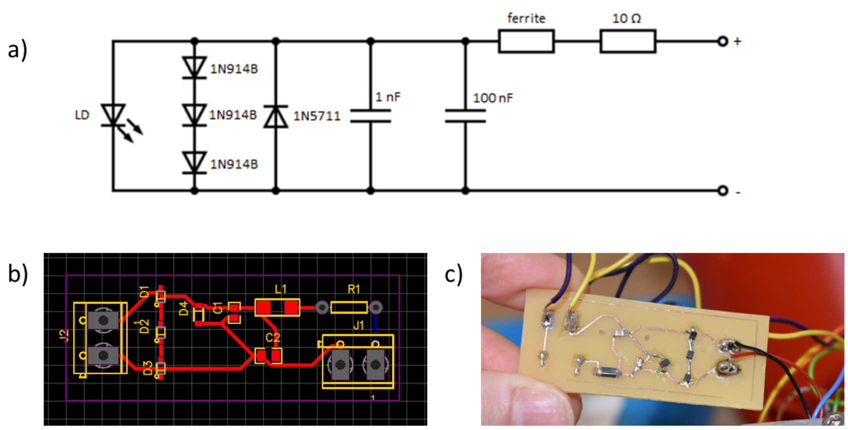

Figure 7: Laser diode protection circuit: a) electronic schematics with specified parts, b) EasyEDA PCB project, c) assembled circuit.

3.3 Current and temperature controller

Current and temperature control is crucial for proper operation of the semiconductor laser. It ensures laser's stability and repeatability of working conditions. The energy gap in a semiconductor changes with temperature and applied current. It results in the shift in the central frequency of the light produced by the laser diode. In this setup, the ITC100 OEM Laser Diode Controller with ITC100D Control and Display Panel (Thorlabs) is used to prevent such changes. Besides energy gap changes, the controller is also used to prevent destruction of the laser diode. Exceeding safe levels of both the current intensity and temperature could cause permanent damage. The ITC100 enables setting the limiting current supply that cannot be exceeded and with the proper temperature control one can avoid overheating of the laser diode.

The elements used for temperature control are:
- ITC100 controller
- Peltier module ($P_{max} = 36$ W)
- electronic temperature sensor (LM335Z)
- base (heat reservoir)
- collimator holder (heat transfer).

The electronic temperature sensor is placed in a special hole put in the aluminium collimator holder so that it is close to the location of the laser diode. By use of a thermally conductive paste it is in a good thermal contact with the metal element. The sensor is a semiconductor based element that changes its output voltage by 10 mW/°C. This voltage is read and analysed by the PID system in the ITC100 controller and appropriate feedback is sent to the Peltier module. The controller enables setting the temperature with accuracy of ± 0.1°C.

The Peltier module is a semiconductor element which converts electrical power into temperature difference. It is designed in such a way that one of its sides is heated while another one is cooled. Applying reverse voltage will change heat transfer direction. In order to cool efficiently, the hot side of the module should be connected to some kind of heat radiator. In presented system, the aluminium base is acting as a heat reservoir. Due to its relatively high volume, large surface of thermal contact with the module and low heat capacity of aluminium, the performance of the base is sufficient for described application. It is worth mentioning that the there is a layer of thermally conductive paste between each side of the module and two aluminium parts; that is the base and the collimator holder. As in the case of electronic temperature sensor, it ensures better thermal contact between the elements.

The ICT100 allows for two ways of current control, that is constant power (CP) and constant current (CC) control. CP control uses the feedback from a monitor diode which is a photodiode incorporated in the laser diode package. It cannot be used in described application of the laser since the injection seeding would change its readout and make it unreliable. This is why the CC control is implemented in the system and the feedback is sent to the ICT100 controller directly from the laser diode's terminals. The ICT100 panel is equipped with external potentiometer which enables easy way for manual current alteration.

3.4 Laser cover with the Brewster window

The cover prevents dust from settling on optical elements, from accidental damage by direct physical contact with laser elements and it provides stable environment for the laser diode operation. The design of the cover made in this project is presented in Figure 8. It is attached to the base with the screws. Both the cover and the base have been designed in CAD software. Based on the technical drawing, the base was turned on the lathe. The cylindrical shape of the base was chosen for the ease of machining as it could be easily turned on a lathe. The cover has been printed on a 3D printer using PLA. This method allows for a lot of flexibility and is very precise. The precision and the possibility of easy implementation of modifications has been important because the narrow tube pointing out of the cylinder does not end flat. It is 'cut' in such a way that the glass plate can be glued to it at the Brewster angle to the axis of laser light incidence. The aim of this is to partially clean the polarization of the output beam by reflecting undesired components. The refractive index of the glass made of BK7 for the wavelength of light used in the experiment is 1.53 [14] and the Brewster's angle at the air-glass boundary is 56.83°. Figure 9 shows the transmission ratios of s and p-polarized light calculated from Fresnel equations:

$$T_{\mathrm{s}} = 1 - \left| \frac{n_1 cos\Theta_{\mathrm{i}} - n_2 cos\Theta_{\mathrm{t}}}{n_1 cos\Theta_{\mathrm{i}} + n_2 cos\Theta_{\mathrm{t}}} \right|^2 \tag{13}$$

$$T_{\mathrm{p}} = 1 - \left| \frac{n_1 cos\Theta_{\mathrm{t}} - n_2 cos\Theta_{\mathrm{i}}}{n_1 cos\Theta_{\mathrm{t}} + n_2 cos\Theta_{\mathrm{i}}} \right|^2 \tag{14}$$

where T_{s} and T_{p} are s and p tansmition ratios, Θ_{i} is the angle of incidence, Θ_{t} is the refraction angle, n_1 and n_2 are refractive indices of the media. The electric field in p-polarized light oscillates in the same plane in which incident, refracted and reflected beams propagate and in s-polarized light the electric field oscillated in the plane which is perpendicular to the mentioned plane. At the Brewster's angle, the transmission of the p-polarized light is almost 100% while the transmission of the s-polarized component is only 83.9%.

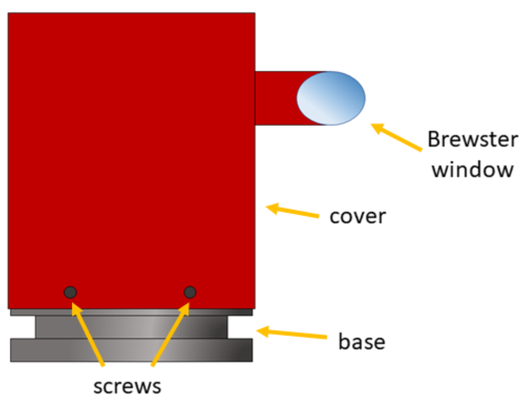

Figure 8: Design of the cover with Brewster window. The cover is attached to the aluminium base with the screws in such a way that their geometrical bases are on the same axis. The pointing end with Brewster window is set on the height that ensures laser light transmission.

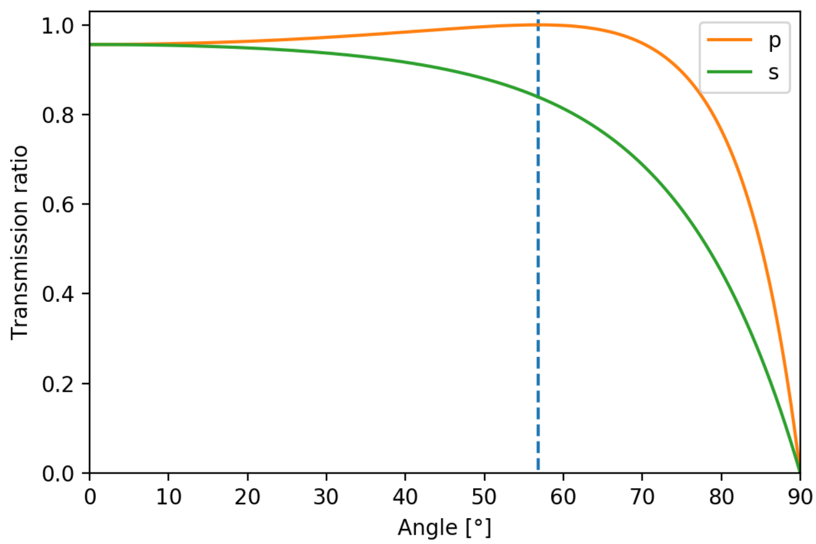

Figure 9: Transmission ratio of s-polarized and p-polarized light passing through a glass plate with refractive index $n = 1.53$ for wavelength equal to 852 nm [14]. Dashed blue line marks the Brewster angle determined for those parameters.

3.5 Optical isolator

The basic scheme of an optical isolator is shown in Figure 10. Let's suppose that the optical isolator is aligned in such a way that its input polarizer (polarizer 1) has a polarization axis in the xz plane, at 45° to the z axis. The input beam should have the same polarization axis, so that the light transmission through the isolator is maximized. Light with proper polarization passes through the first polarizer and the light whose polarization axis is orthogonal gets absorbed. Then the light passes through the Faraday's rotator, where the polarization plane is rotated by 45° in the clockwise direction. The polarization axis at the output polarizer (polarizer 2) is set in the xz plane, at 0° angle to the z axis. If some part of the light is not rotated properly, the orthogonally polarized component does not pass through this polarizer but is absorbed instead. The Faraday rotation phenomenon ensures non-reciprocal optical propagation. It means that the light propagating in the reverse direction (e.g. retroreflected) is effectively blocked. At first, that light passes through the output polarizer and only horizontally polarized light passes through. Then the polarization plane is rotated by 45° in the Faraday rotator in clockwise direction as seen by the incoming light. Now its polarization axis is exactly perpendicular to the axis of the first polarizer and cannot propagate further. The retroreflected light is undesired since it could cause phase noises or sfifts in emitted light frequency once it gets to the laser diode cavity. Optical isolator works also as a polarization cleaning device since the output beam has well defined polarization plane.

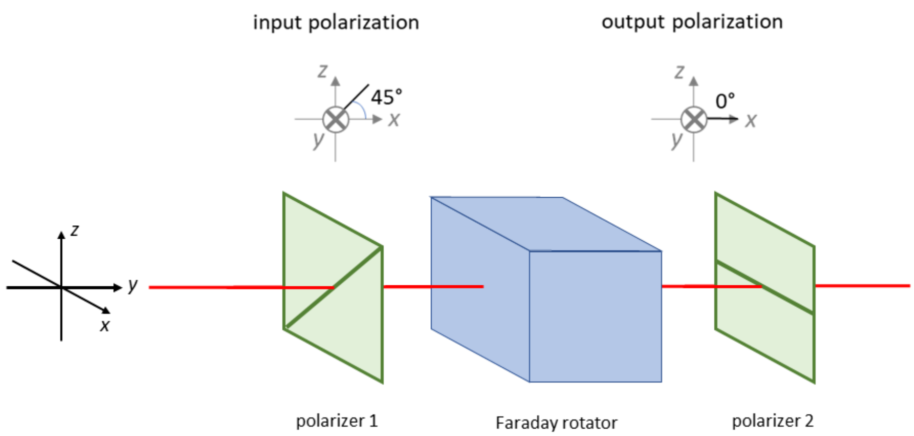

Figure 10: Scheme of optical isolator structure. The input beam with polarization axis in the xz plane, at the angle of 45° to the z axis, passes through the first polarizer. After passing through the Faraday rotator the polarization of the light is rotated and it is at 0° to the z axis, hence it can pass through the second polarizer.

For high power lasers, this kind of optical isolator is not the best option as it may break or work improperly due to high absorption of light that is blocked. It is better to use an isolator with 'rejection' ports through which the undesired light can escape instead of being absorbed. Instead of usual polarizers, polarizing beam splitting optics is used. Figure 11 shows an example of such an isolator. PBS1 allows for light transmission with polarization axis in the xz plane, at 45° to the z axis, the remaining part of the beam is reflected in orthogonal direction and escapes through the port. PBS2 separates the part of the beam which was improperly rotated in the Faraday rotator. It escapes through the port which is later used for injection seeding. This kind of optical isolator is used in this project (model IO-3-850-HP from Thorlabs) because by passing the seeding beam in the reverse direction through one of the 'rejection' ports, injection seeding of a laser diode is possible.

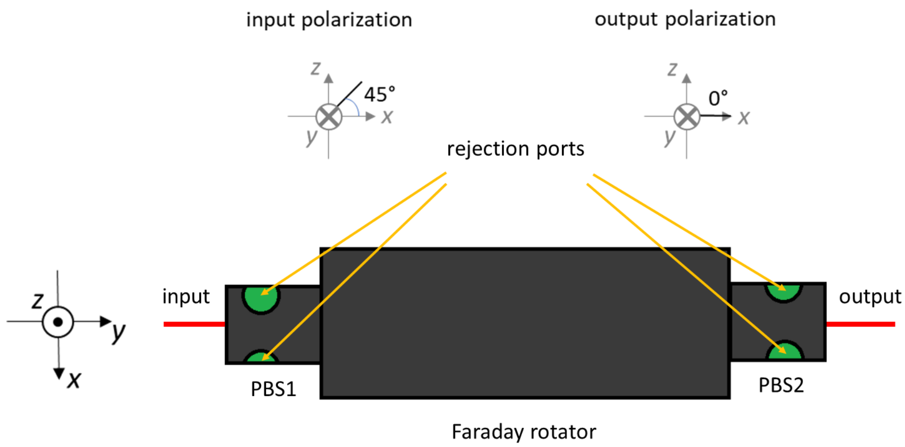

Figure 11: Scheme of the structure of optical isolator with 'rejection' ports. PBS1, PBS2 - polarising beam splitters that allow for light transmission with polarization axis in xz plane, at 45° and 0° to the z axis correspondingly.

3.6 Injection seeding optical setup

An important step in the injection locking procedure is overlapping the incoming seeding beam from the master ECDL laser with the outcoming beam from the slave. It could be done by placing a glass plate at the proper angle so that it reflects some of the master's light to the laser diode. The disadvantage of this method is that a high intensity master's beam should be directed at the glass plate in order to obtain seeding beam of a sufficient power. Due to low reflection transmission ratio, the slave should also be placed near the master laser light source so that there is no need of coupling the light into the fibre as it results in power loss.

Injection locking requires the seeding beam to have the same polarization as the output beam of the slave. Both beams should be mode-matched as well as possible. Optical scheme presented in Figure 12 enables optitmization of those parameters. In this work, the seeding beam is injected into the slave laser cavity through an optical isolator. The injection is possible because of the construction of the isolator (see 3.1.5). It uses polarizing beam splitting optics to reject retro-reflected light as a mean to protect the laser diode. The output of the polarizer, called "rejection port", is used as an input port for the seed beam - the light can be sent in the reverse direction directly into the laser diode. PBS2 (Figure 11) lets the light with the horizontal (p) polarization pass through and reflects the light with the vertical (s) polarization. Rejected beam from PBS2 is then coupled to coupler 2 (Figure 12). Then a fraction of master's light beam was coupled to coupler 1 and its transmission through an optical fiber was maximized. Fibre optic couplers are equipped with a collimator to maximize the intensity of light beam that is sent through an optical fiber by changing its size on the fiber's input. After passing through the fiber, the litgh outcouples with some angle of propagation characteristic for this fiber. This way, by outcoupling master's light beam with coupler 2, the same one which was used for slave's rejected beam coupling, the spatial distribution of those beams becomes very similar. The advantage of this solution is that it allows for relatively easy approximate mode matching, which is sufficient in most applications. It does not require a camera to monitor profiles of both beams which would be problematic since the beams propagate in different directions.

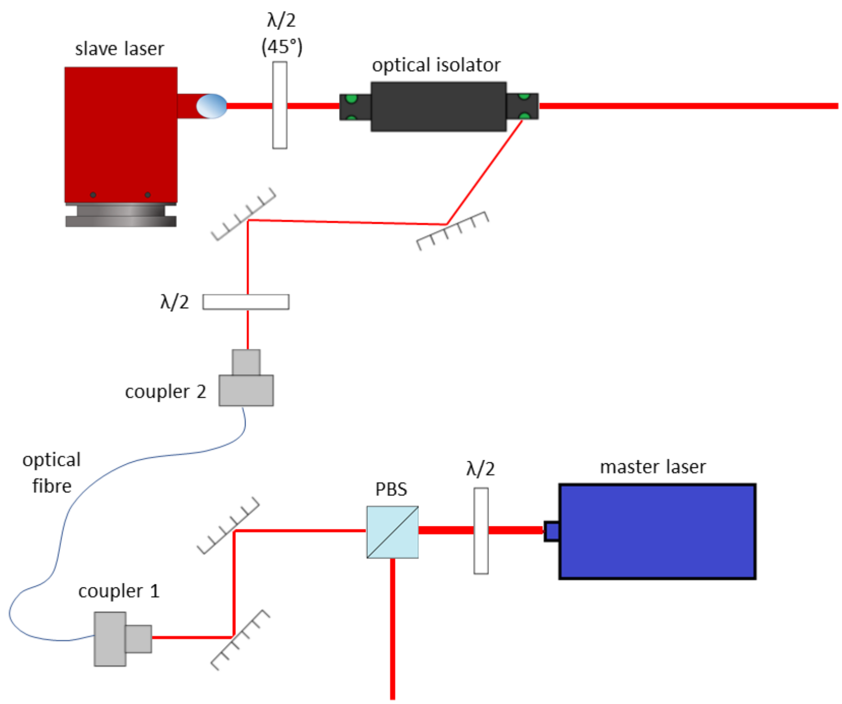

Figure 12: Injection seeding optical scheme; coupler 1 and coupler 2 - fibre optic couplers, PBS - polarising beam splitter, $\lambda/2$ - half-wave phase plate.

Another requirement, polarization plane matching of seeding and slave's output beam, is handled with the use of polarising optics. Polarizing optics is also used for transmission maximization in the system. The master laser beam is split into two beams with a polarising beam splitter (PBS). The half-wave plate between PBS and the master is used for seeding beam power control. The beam is then coupled into a single-mode polarization maintaining optical fibre through coupler 1 and outcoupled with coupler 2. Using polarization maintaining otpical fiber prevents power fluctuations in the system by preserving linear polarization of the light. An optical fiber without this function is susceptible to polarization changes due to environmental factors. The polarization plane would then change in time as well as the splitting ratio on the PBS inside the optical isolator. The half-wave plate is introduced after coupler 2 to control the polarization plane of the seeding beam, which should be the same as the "rejected" beam's, so that the transmission is maximized once it passes through the optical isolator. The laser has been constructed with acceptance of some standards, one of which is that the laser diode in the slave laser is set in such a way that the polarization of the output beam is in the horizontal plane. However, the optical isolator rejects all the light whose polarization plane is different than 45°. This is why another half-wave plate is introduced in the setup to maximize the slave beam's transmission through the isolator and to rotate the angle of the seeding beam's polarization plane so that it is horizontal.

It is not enough to maximize the seeding beam transmission through an optical isolator to optimize its alignment. It is more important that the seeding beam has similar spatial distribution of light intensity in the cross section as the slave's output beam. Since it would be hard to observe the profiles of the beams directly, another method of optimization has been introduced besides the method of overlaping two beams using fibre optic couplers. The slave laser has been set slightly below its threshold and the seeding beam has been overlapped with slave's beam with some efficiency. After injection seeding of the laser diode working near the threshold, the output power of the slave increases. The seeding beam alignment has been optimized by maximizing slave's power gain.

4 Measurements and results

4.1 Characterization of the laser operation

As it has been mentioned before, the laser diode is in fact a semiconductor crystal with a p-n junction. The population inversion in its active medium is achieved through pumping with electric current. Pumping power must reach certain threshold value for the lasing to be observed. Minimum current for which this power is obtained is called the threshold current. Figure 13 presents optical power dependence on the current measured for the laser diode used in the project. The threshold current is determined as the value of the current at the intersection of fitted lines; the one with smaller slope, blue, is in the region of spontaneous emission domination, and the one with larger slope, orange, is in the region of stimulated emission domination (where the lasing occurs). The equation of the blue line is P_{blue}=0.02I-0.20, and the equation of the orange line is: P_{orange}=0.79I-17.90. According to the laser diode datasheet, the threshold current is between 20-40 mA. Experimental data show that threshold current is (23.0 ± 0.7) mA for T=15.9°C, in agreement with the expected value.

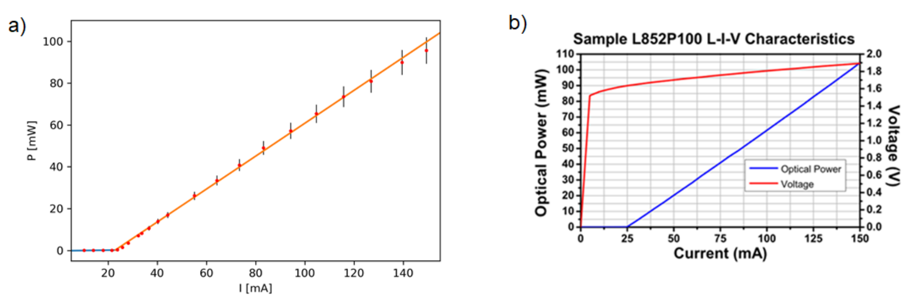

Figure 13: Optical power against laser diode current; a) experimental data, b) data provided by the producer.

Another behaviour characteristic for semiconductor diode is mode hopping. The frequency of light emitted by a laser diode changes both with temperature and current. One could expect linear change when those parameters are increased or decreased but in reality it is not observed. Since laser diode has a cavity, certain waveleghts are enhanced depending on the cavity length. As the working conditions change, some other mode might become more advantageous which results in a step change of the wavelength. Figure 14 presents the wavelength dependance on temperature and current measured with a wavemeter. According to the results, an enhanced light mode changes each $3.5 - 4$°C for the temperature dependance and approximately each 35 mA for the current dependance. It is important to realise that these step changes in light frequency occur since they can affect the injection locking. If the parameters of the laser are set on the border of two modes, it can easily fall out of the injection locking window.

T[°C]	Freuency change rate [GHz/mA]
17.0	-1.12
20.9	-1.14
23.9	-1.16

Table 1: Frequency change rate within one longitudinal cavity mode for current range 120-150 mA measured for three different temperatures.

The data in Figure 14 show some unexpected behaviour of the laser diode. The mode of light changes within one 'step' of wavelength/current and wavelength/temperature dependancy. We would expect the wavelength to change linearly within one lonigtudinal cavity mode but instead we observe some discontinuities. Perhaps the laser diode working regime is not monomode resulting in additional modes appearance. Nevertheless, the wavelength change rate is in agreement with data from AlGaAs lasers characterization [12]. The rates of the wavelength change measured for the laser diode used in this project are presented in Figure 14 and Table 1.

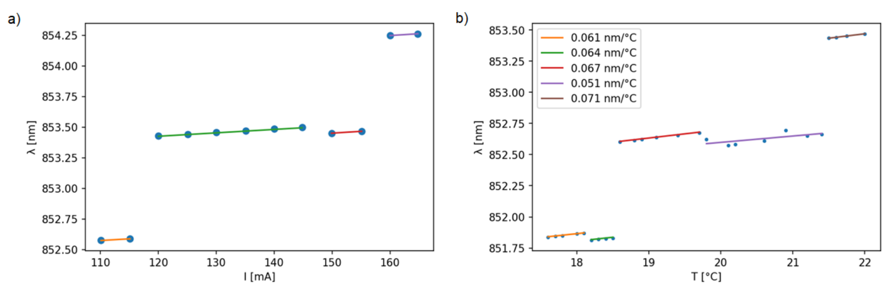

Figure 14: Wavelength dependence a) on the laser diode current for T=23.9 °C, b) on the temperature for the diode current set to 135 mA. The slope in each range is shown in the legend of the plot.

4.2 Injection locking near the threshold

The influence of injection locking is the most visibly observed when the laser diode operates near threshold current. As it was predicted by Li [3] and later confirmed by experiment [4], the gain change increases and the threshold current value decreases when the laser diode is injection locked. Figure 15 shows the graphs of optical power against the laser diode current for different powers of injected light, ranging from 0 mW to 1.090 mW. The power of emitted light increases as the injected light power increases. The curves get smoother and the transition from spontaneous to stimulated emission is more continuous. Based on the data shown in Figure 15, the threshold current has been determined for each data set using the same method as for data from Figure 13. Values of the threshold currents for different injection powers are shown in Figure 16. At the begining, the threshold current decreases rapidly, but for powers greater than 0.6 mW, the change is barely noticeable, threshold current possibly reaches some asymptote value. All of these measurements have been made for laser diode temperature equal to 19.2°C.

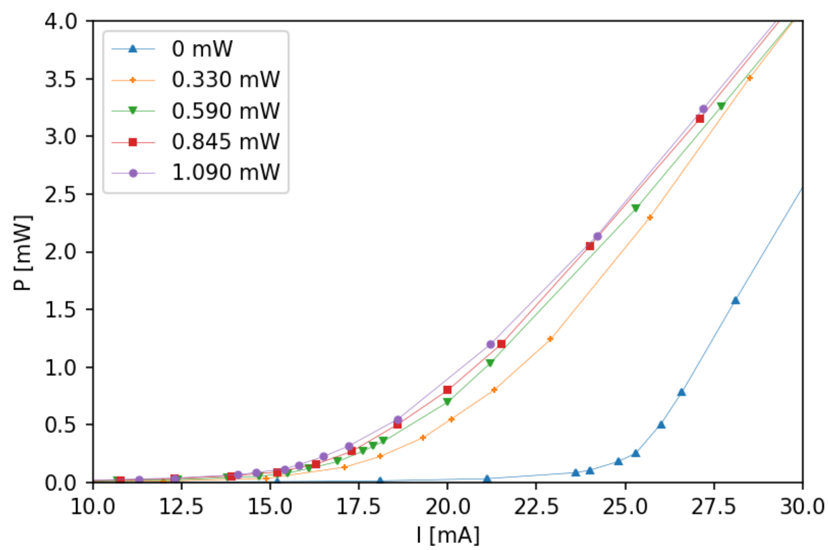

Figure 15: Optical power against slave laser current for different powers of injected light.

Another complementary measurement has been made to observe the gain change due to injection locking. The laser diode current and temperature have been set at constant values of 22.4 mA and 19.2°C, respectively, such that the laser initially operated below the lasing threshold. The results are shown in

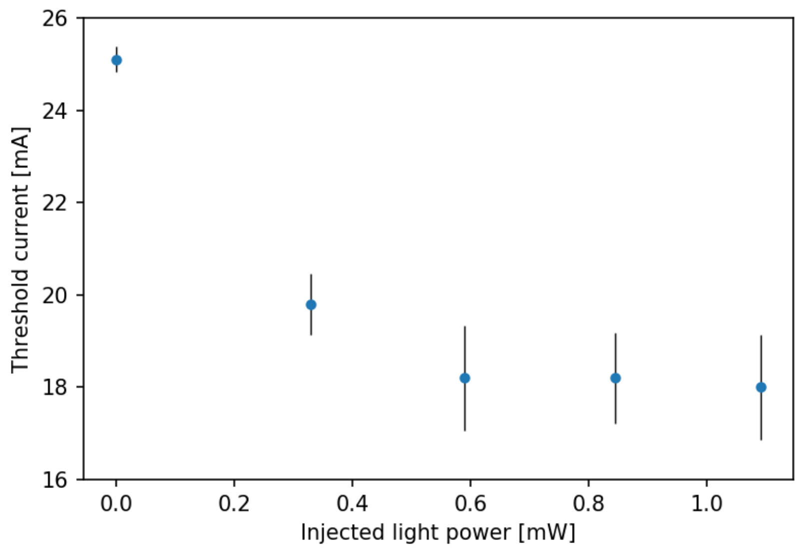

Figure 16: Laser diode threshold current dependance on the injected light power.

Figure 17. Two parameters have been measured for each seeding power: P_{refl} - power of the light reflected from the laser diode when no current was supplied, P_{out} - output power of the seeded laser diode. The gain change was calculated from the formula below:

$$G = (P_{\mathrm{out}} - P_{\mathrm{refl}})/P_0 \tag{15}$$

where P_0 is the power of light emitted by free-running laser. The gain increases with increase of the injected light power but the increase rate declines for its highest values. The part of the graph from 2.0 to 3.0 mW flattens and it might reach some asymptotic value later on. The laser diode was not seeded with powers exceeding 3.0 mW because of precautionary measures.

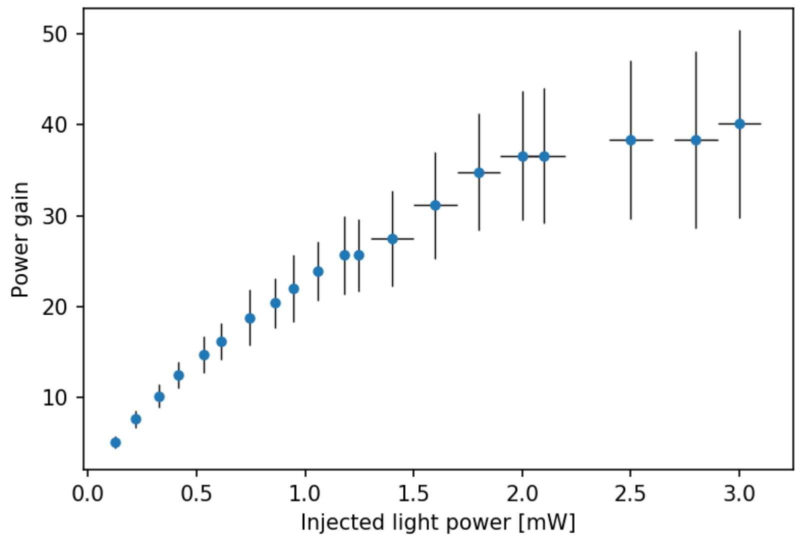

Figure 17: Power gain vs injected light power for the laser diode current biased below the threshold, I=22.4 mA.

4.3 Injection locking above the treshold

4.3.1 Injection locking conditions for the seeding beam stabilized to an atomic transition

The success of injection locking depends on parameters such as the frequency difference between the master and the slave, the seeding beam power and the efficiency of coupling of the seed into the laser diode cavity. Injection window is the range within which a parameter can be altered so that the injection locking still occurs. Figure 19 presents current injection windows for various powers of injected light, when the master laser was stabilized to an atomic transition in ceasium atom. The injection success was verified with two tools: wavemeter and Fabry–Perot cavity. Figure 18 presents a scheme of Fabry–Pérot cavity. It consists of two mirrors having the same optical axis, one of which is stationary and the other one is glued to the a piezoelectric element that enables movement along mirror's axis along which the laser light is introduced into the cavity. The cavity length changes linearly with voltage applied to the piezoelectric element. Because of interference of light inside the cavity, only the wavelengths equal to:

$$\lambda = \frac{2l}{n} \tag{16}$$

are enhanced, where l is the cavity length and n is some natural number. A photodiode measures signal of the output beam which is proportional to the light intensity in the cavity. The spectrum is observed on the oscilloscope as the voltage change on a photodiode detector. Since the signal from Fabry–Pérot cavity used for this measurment is not scaled to the light frequency, the wavemeter was used to check with very high accuracy whether injection locking resulted in slave's light having the same frequency as the master's light.

The criteria for confirmation of successful injection locking were: single mode spectrum shape as in Figure 19f), h) and j), and exactly the same frequency of observed single mode as the master's light frequency. The situation when the central frequency of the most intense mode was the same as master's but some additional modes appeared, as it was the case for Figure 19c), did not meet the criterion.

What's interesting, falling out of the injection window because of too low current was very rapid and resulted in a complicated, multimode spectrum as in Figure 19d), e) and i). What's more, for lowering current below some point, the injection was successful only if the seeding was continuous and unperturbed. Once the seeding beam was blocked and unblocked again, the laser diode was no longer injection locked. The situation looked different for increasing the current. Falling out of the injection window from this side resulted in additional modes appearing in the spectrum as in Figure 19c), g) and k). This process was gradual and further increase of the current caused the rise in intensity of newly appearing modes.

When it comes to the characterization of the injection locking windows, shown in Figure 19a), some of their properties can be described. The width of these windows increase with the increase of injected light power. It means that it is harder to fall out of the injection window and that the injection is possible for larger frequency differences between the master and free-running slave. Another observation is that the windows 'shift'; their central current value changes. Possibly injecting more light from the master to the slave's cavity results in changes of the laser diode properties like the refractive index of the active medium.

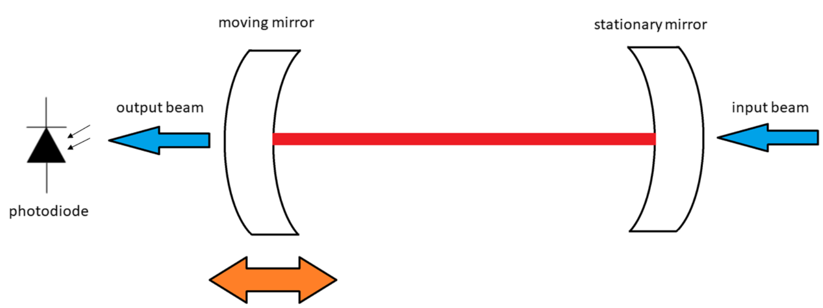

Figure 18: Fabry–Pérot cavity structure scheme.

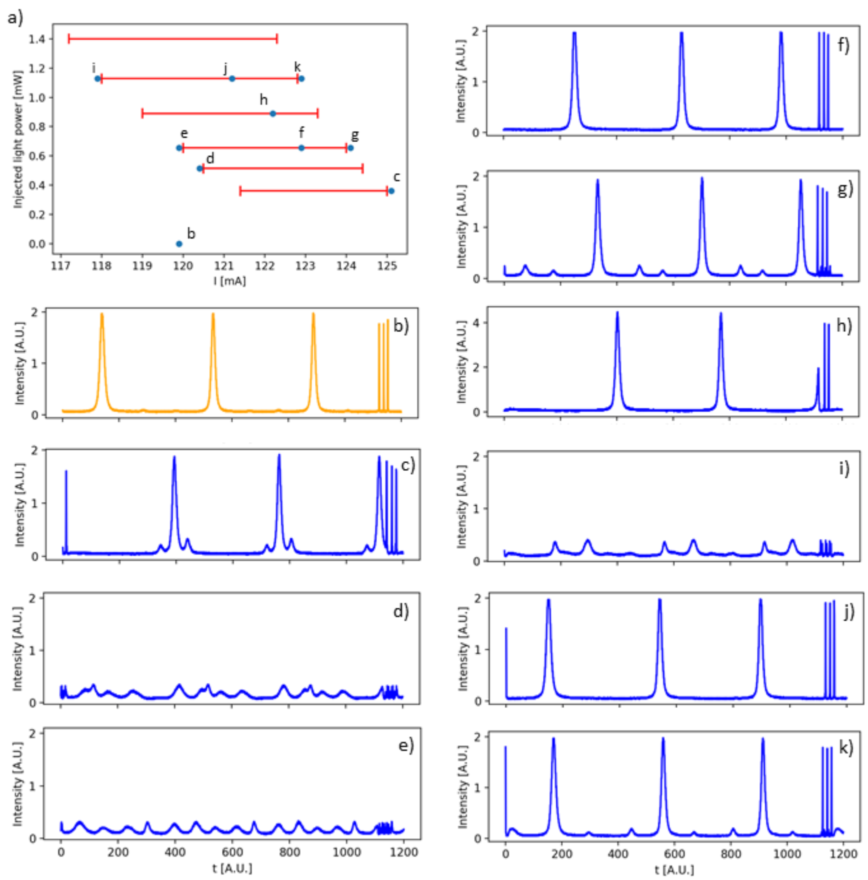

Figure 19: Seeding windows for different powers of injected light and light spectra observed in Fabry–Pérot cavity. Blue dots correspond to parameters for which the spectra b)-k) were measured. The measurment for point b) was made when no injection seeding was applied.

4.3.2 Seeding beam frequency scanning

Another part of experiment was to check the injection condintions for the case when the frequency of the seeding beam is scanned. The scanning was realised by changing the current of the master laser, which was done remotely with use of DigiLock, a commercial device controlling the master laser current and temperature. While scanning seed beam frequency, the saturated absorption spectrum of the slave's beam was observed.

The saturated absorption spectroscopy is a method for surpassing the Doppler limit while observing absorption spectrum of a gas. In this experiment, a cell with caesium gas was used. Basic scheme of the saturated absorption spectroscopy can be found in Figure 20. It uses a pumping beam, a probe beam and sometimes also a reference beam, and all of these beams pass through the cell. If the light frequency is in resonance with some allowed atomic transition, the absorbtion occurs. The pumping beam has the highest intensity and it is meant to saturate the light absorption along its path in the atomic vapor. The probe beam, with lower intensity, propagates in the opposite direction. Since the absorption is saturated by the pumping beam, the light of the probe beam at resonance frequencies that correspond to atomic transitions, is no longer absorbed. Because of the Doppler effect, moving atoms perceive counterpropagating beams differently and the probe beam absorption will be decreased only in the frequency regions of natural transition linewidths and the Doppler limit is effectively surpassed. The probe beam's transmission is detected after passing the vapour cell and it could already serve as the spectroscopic signal. One could observe dips in Doppler-broadend absorption spectrum which occur in the regions of natural linewidth of atomic transitions. The reference beam can be added by splitting the probe beam into two separate beams that propagate on parallel paths. The reference beam passes through the part of the cell which is not affected by the pumping beam, thus it gets absorbed when its frequency is resonant with the transitions in the atomic vapour. Introducing reference beam into system and comparison of its transmission with the probe beam's transmission makes the method more sensitive.

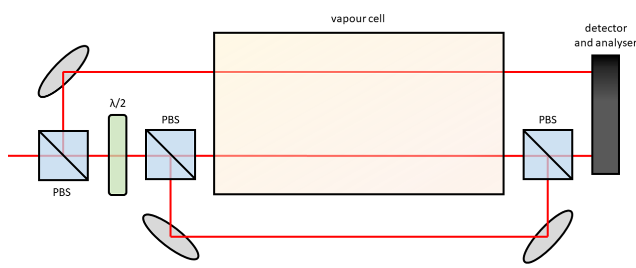

Figure 20: Example scheme of saturated absorption spectroscopy setup.

Figure 21 shows a saturated absorption spectrum of the master laser light and the slave laser light while the frequency of master light is scannned through D2-line caesium transitions. Even though there are only three possible transitions from F=3 ground energy level, and also only three transitions from F=4 ground energy, in both cases the total of six peaks is observed. Those are the crossovers and their existance can be explained by the Doppler effect. Since the pump and the probe beams are counterpropagating, their k-vectors have opposite signs. Atoms with non-zero velocity in the axis of light propagation will perceive them differently. When the laser light frequency is equal to the average energy of two atomic transitions, some atoms will be in resonance with both beams which results in decreased absorption.

In Figure 21a) the spectroscopic signal is very clear and the background noise does not change much within the scanned frequency span. Spectra from Figure 21b) have higher and non-uniform backround noise. It results from not well optimized settings of the saturated absorption spectroscopy signal analyser. It could be avoided by changing probe to pumping beam intensity ratio and the offset of the dark noise of the photodiode detector.

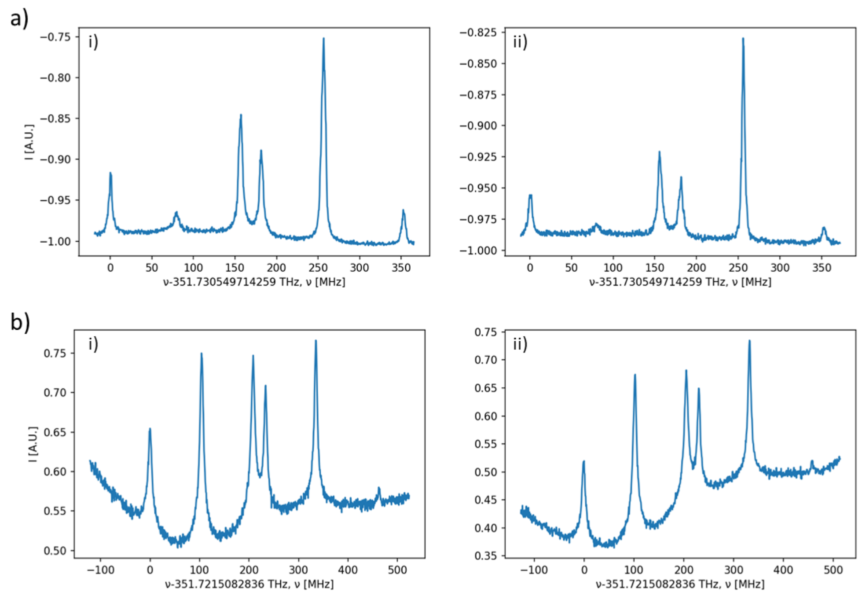

Figure 21: a) The saturated absorption spectrum of D2 line transitions of caesium from $F = 3$ ground energy level for i) ECDL laser, ii) slave laser locked to the ECDL light amplified in a tapered amplifier. Peaks from left to right: transition to $F' = 2$, crossover (c.o.) of transitions to $F' = 2$ and $F' = 3$, transition to $F' = 3$, c.o. of transitions to $F' = 2$ and $F' = 4$, c.o. of transitions to $F' = 3$ and $F' = 4$, transition to $F' = 4$. b) The saturated absorption spectrum of D2 line transitions of caesium from $F = 4$ ground energy level for i) ECDL laser, ii) slave laser locked to the ECDL light amplified in a tapered amplifier. Peaks from left to right: transition to $F' = 3$, c.o. of transitions to $F' = 3$ and $F' = 4$, transition to $F' = 4$, c.o. of transitions to $F' = 3$ and $F' = 5$, c.o. of transitions to $F' = 4$ and $F' = 5$, transition to $F' = 5$. The injection light power for presented data is 1 mW.

The seeding windows for the current change were specified based on the visual assessment of the quality of the saturated absorption spectroscopy spectra for D2 line transitions in caesium from the F=4 ground state level. Three spectra realised for injection light power equal to 0.890 mW are shown in Figure 22. These spectra are broadened because of incorrect settings of signal amplification. The ratio of reference to probe beam signal was not adjusted

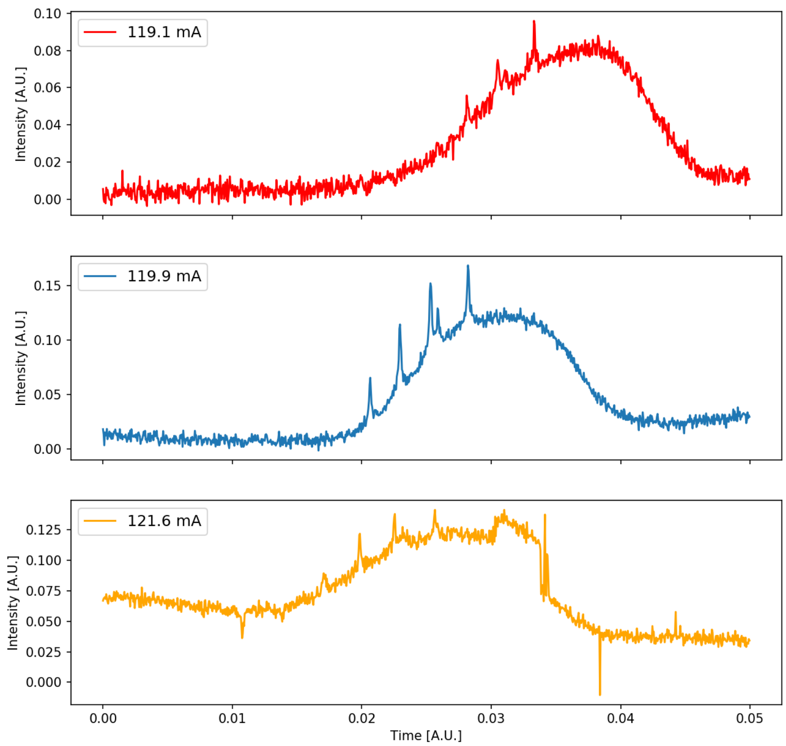

Figure 22: Saturated absorption spectra of seeded slave laser beam while the master laser frequency is scanned through caesium D2-line transitions from F=4 ground state. Power of injected light is 0.890 mW.

resulting in improper signals subtraction. For the current set to 119.9 mA, the five absorption peaks are well resolved so this parameter value was recognized to be within injection window. For currents equal to 119.1 mA and 121.6 mA, some of the peaks are comparable to or indistinguishable from the noise. These spectra are of low quality, what means that the injection locking is not fully successful and that these values of currents are outside the injection window. As expected from the theory, injection windows are wider for higher powers of injected light what is oberved in Figure 23. Part b) of the figure shows also the comparison of injection windows widths when the seed beam frequency is scanned and locked to the atomic transition. The theoretical description of injection locking phenomena states that the injection window is proportional to the intensity of injected light, $\Delta\omega \propto \sqrt{I_1}$. By assuming that the power of the injected light is proportional to its intensity and that the frequency of the free-running slave laser changes linearly with current and no mode hopping occurs for supplied current range, another relation can be derived, which states that $\Delta I \propto \sqrt{P}$, where P is the optical power of the injected light. This relation is fitted in Figure 23b. Injection locking is successful for wider current intensity range when seeding beam frequency remains constant. It is a direct proof that the frequency change between the master and free-running slave light influences the injection locking.

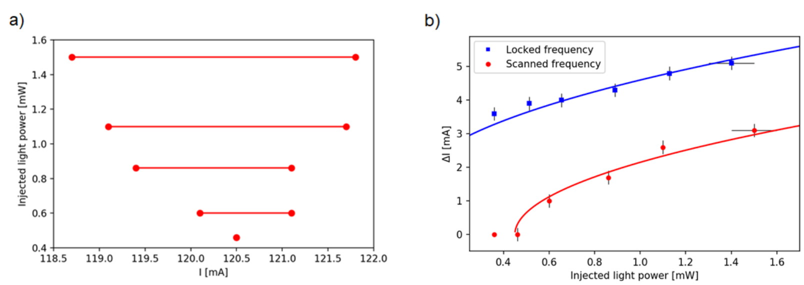

Figure 23: a) Seeding windows for different powers of injection seeding when the seed beam frequency is scanned. b) Comparison of injection locking windows widths when the seeding beam frequency is stabilized to an atomic transition (window widths determined from data in Figure 19, blue squares) and when it is scanned (window widths determined from data in a), red dots).

5 Conclusions

A semiconductor laser has been successfully assembled, resulting in creation of a stable laser light source. It has been shown that the frequency of the laser diode used in this project can be successfully injection locked to the master laser light for injected light power smaller than 0.4 mW. The injection locking is also possible when the frequency of the master laser is scanned and it is enough to use 0.5 mW of injected light power for the scanning range of almost 400 MHz. The experimental results are in agreement with the theory and results of other groups. It confirmed that the injection locking window and power gain increases while the threshold current decreases with the increase of injected light power.

5.1 Future work

This work presents how to perform the injection locking resulting in the slave laser light having exactly the same frequency as the master laser light. Nevertheless, this is not sufficient to obtain the repumping light, since the desired frequency has to be shifted from the master light frequency by ~9.2 GHz. The next step is to implement the laser diode current modulation and injection locking of the generated sideband like in ref. [8]. The procedure uses RF generator, which produces signal of frequency equal to the desired frequency difference between the master and the slave light. The RF signal is added to the direct current component through bias tee and sent to the slave laser diode. The modulation causes creation of the sidebands in the spectrum with frequencies equal to $\omega_0 \pm n\omega_m$, where ω_0 is the carrier wave frequency, n is some natural number and ω_m is the RF signal frequency. Figure 24 presents the results of laser diode current modulation with 3 GHz frequency RF signal as obtained in ref. [15]. The b) part of the Figure 24, the +1 and -1 order sidebands are clearly visible. Such a sideband could be then injection locked to the master laser light frequency. The required optical setup is presented in Figure 25. In this project, an RF signal generator (EVAL-ADF5355 from Analog Devices) and a bias-tee (ZX05-153LH-S+ from Mini-Circuits) will be used.

The data from Figures 22 and 23 show saturated absorption spectroscopy spectra and injection windows for frequency scanned through D2 line transition in caesium from F=4 ground level. In the laser cooling procedure, the cooling beam is resonant with F=4 to F'=5 transition. The experimental results show that the slave laser frequency can be injection locked to the cooling beam frequency in a stable manner. In this work, the master laser will be used as the cooling beam source and the slave will be used as source of repumping beam. After the slave laser diode current modulation with the RF signal of frequency equal to 9.192380678 GHz, the -1 sideband will be locked to the master laser frequency resulting in locking slave carrier wave frequency at the repumping transition.

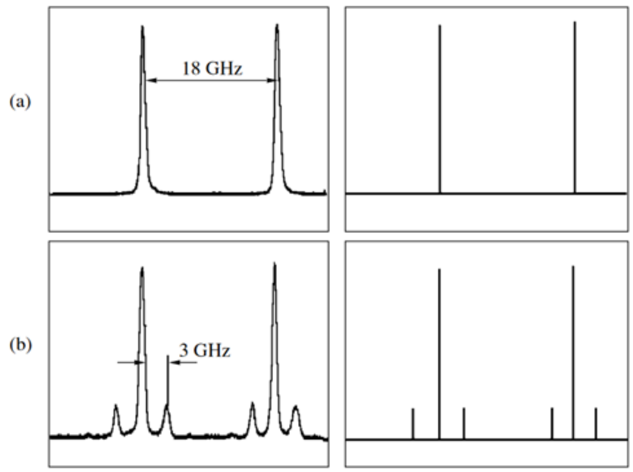

Figure 24: Spectrum of slave laser light measured by Fabry-Perot cavity when a) laser diode current is not modulated, b) when laser diode is modulated with RF signal of 3 GHz. Left side - experimental results, right side - theoretical predictions. Figure adapted from [15].

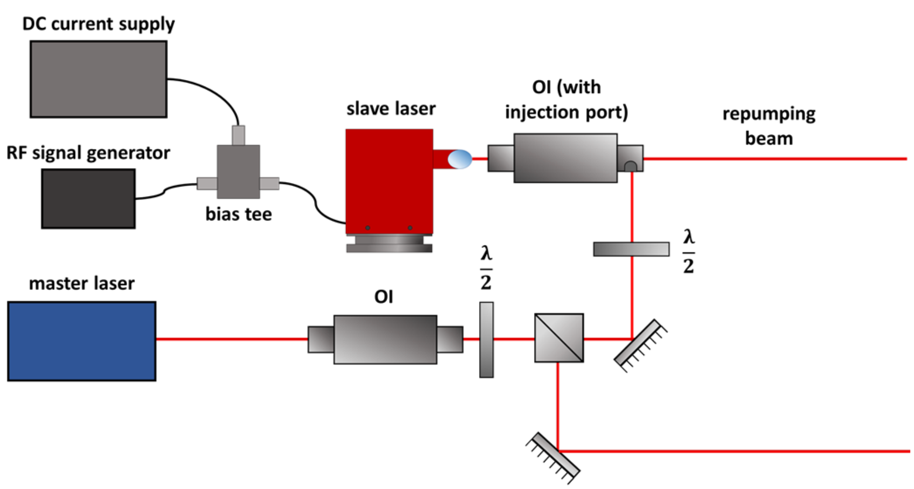

Figure 25: Scheme of the final optical setup, OI - optical isolator. The current of the slave laser is modulated with RF signal. RF and DC (direct current) components are summed up with use of bias tee. Beam generated by the slave laser has frequency of the repumping light.

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